Fluid Physics

8.292J/12.330J

Problem Set 1

Due Wednesday, 18 February

1. Consider a planetary atmosphere consisting of an ideal gas of gas constant R in which the temperature decreases linearly upward from the surface:

$$T = T_s - \Gamma z,$$

where T_s is the surface temperature and Γ is the gradient of temperature with altitude (sometimes called the *lapse rate*). Assume that the atmosphere is at rest within a gravitational field characterized by a uniform gravitational acceleration g, and that the surface pressure has the value p_0. Derive an expression for the atmospheric pressure as a function of altitude. At what altitude does the pressure vanish? Why?

2. A cube of incompressible fluid of density ρ_c with massless walls is suspended within another incompressible fluid of density ρ_a by means of a massless string attached to wall. The entire system is at rest within a gravitational field of uniform gravitational acceleration g. The length of any side of the cube is Δx. Derive an expression for the net force acting on the massless string. Now the string is cut. Find the acceleration of the cube at the moment the string is cut.

3. (Extra Credit) A completely rigid cylinder in a uniform gravitational field is completely filled with an Euler fluid of constant density ρ which is at rest. Within the fluid is a small balloon filled with an ideal gas. This balloon is attached to a string whose volume is negligible, by which means the balloon's vertical position can be changed. The tension of the balloon's material can be neglected, so the gas may be taken to be in pressure equilibrium with the adjacent Euler fluid. At the initial time, t_0, the fluid pressure at the position of the balloon is p_0 at an altitude of z_0 above the bottom of the cylinder. The balloon is then slowly raised or lowered to a new position z_1 at time t_1. Describe the pressure distribution in the fluid at time t_0 and at time t_1.