

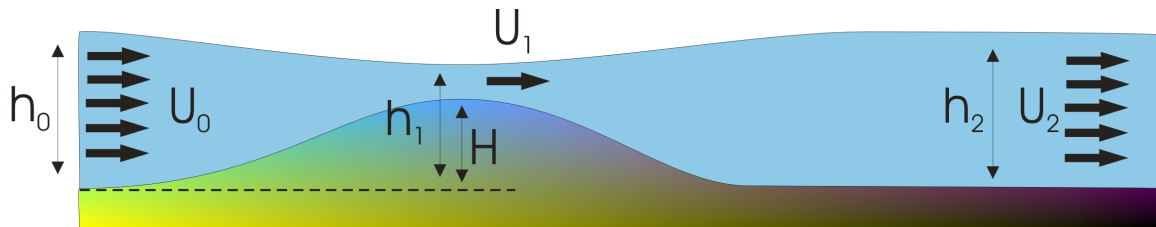
Fluid Physics
8.292J/12.330J

Problem Set 2

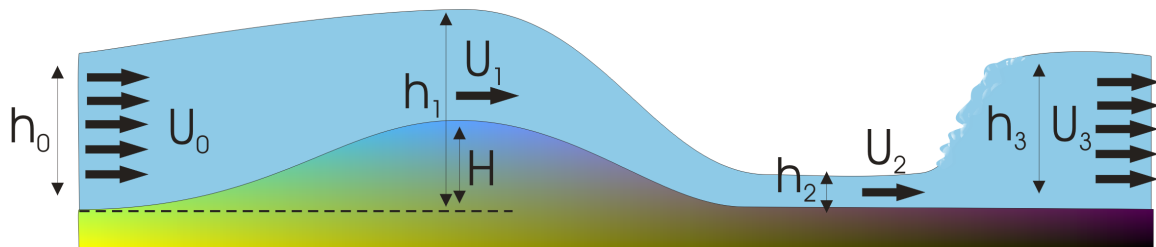
1. A shallow layer of Euler fluid of depth h_0 in a uniform gravitational field, given by acceleration g , flows from left to right with velocity U_0 and encounters a smooth bump of height H . The bottom returns to its initial height rightward of the bump. Consider the flow to be two-dimensional (i.e. without variation in the horizontal direction normal to the flow). With one exception noted below, the flow can be considered steady. We define a nondimensional number F (called the Froude number):

$$F \equiv \frac{U_0^2}{gh_0}$$

The flow can assume one of two configurations, as illustrated below:



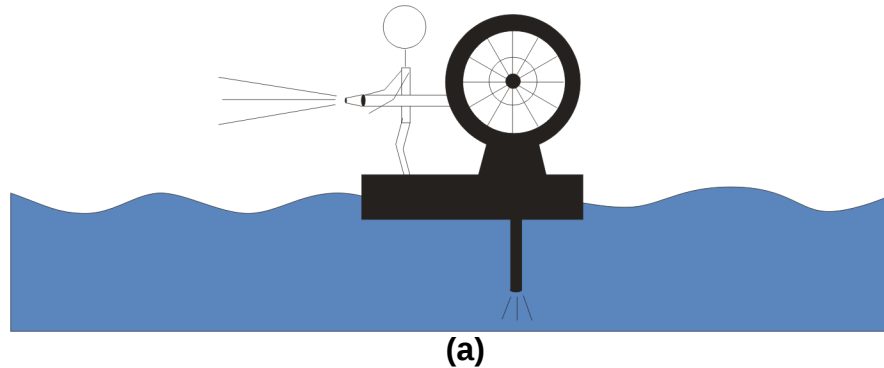
(a)



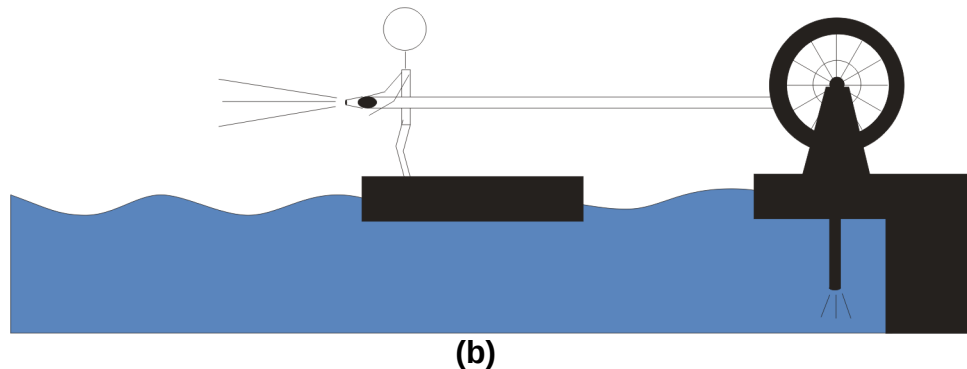
(b)

In regime (b), a hydraulic jump (a stationary bore) occurs downstream of the bump; the flow near it is turbulent and dissipative.

- a.) For each of the two regimes, find expressions for h_2 and U_2 .
 - b.) For regime (b), also find expressions for h_3 and U_3 .
 - c.) Find approximate expressions for h_1 and U_1 in the limit that F is very small.
 - d.) Find approximate expressions for h_1 and U_1 in the limit that F is very large.
 - e.) Extra credit: For regime (b), find an expression for the net horizontal force on the bump, being sure to indicate which direction it acts.
- 2a.** A barge, floating on a pond, is equipped with a pump which takes water from the pond and pumps it out through a hose fitted with a nozzle (a). The water may be idealized as an Euler fluid for this problem. The density of the water is ρ , the cross-sectional area of the end of the nozzle is A , and the velocity of the water as it exits the nozzle is V . Find the force on the barge.



- 2b.** Consider the same problem as in 2a, but this time the pump is located on a dock (fixed to the ground; see (b)). For the purposes of this problem you may neglect gravity and assume that there are no tensile forces acting on the hose. What is the force on the barge? How might your answer change if you allow for friction of the water flowing through the hose?



3. (Extra credit) Consider the problem described in problem 2b, but allow for gravity, so that the hose sags. Assume that the hose has infinite tensile strength (it cannot be compressed or extended along its length) but has no resistance to bending. Neglect friction of the flowing water. What is the force on the barge? (Note: to get a quantitative answer, you might want to consider special configurations of the hose.)