# Thermal wind and temperature perturbations

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October 15, 2002

Following the work of Bretherton (1966), we showed in class that a boundary potential temperature perturbation behaves in the same way as a delta function perturbation of potential vorticity would just inside of the boundary. By integrating the definition of pseudo-potential vorticity over an infintesimal region adjacent to one of the horizontal boundaries, we showed the mathematical equivalence of a boundary potential temperature anomaly and a delta function $q_p$ anomaly just inside the boundary. In the case of a lower boundary, the temperature anomalies behave like delta function $q_p$ anomalies of the same sign; at the upper boundary, they behave like $q_p$ anomalies of the opposite sign.

It can be useful to have an illustration of how boundary $\theta$ anomalies behave. The presence of such an anomaly in a quasi-balanced fluid *implies* the presence of an anomalous circulation by the thermal wind relation. Recall that the thermal wind equation is valid for geostrophic motions in a hydrostatic fluid. By differentiating the hydrostatic equation,

$$\frac{\partial \phi'}{\partial p} = -\frac{\theta'}{\pi}, \tag{1}$$

with respect to $x$ and differentiating the definition of geostrophic wind,

$$v'_g = \frac{1}{f}\frac{\partial \phi'}{\partial x}, \tag{2}$$

with respect to $p$, the thermal wind equation relating vertical wind shear to horizontal gradients of potential temperature may be obtained:

$$\frac{-1}{f\pi}\frac{\partial \theta'}{\partial x} = \frac{\partial v'_g}{\partial p}. \tag{3}$$

Similarly,

$$\frac{1}{f\pi}\frac{\partial \theta'}{\partial y} = \frac{\partial u'_g}{\partial p}. \tag{4}$$

So wherever there are gradients of $\theta'$ on a horizontal boundary (e.g., heading into or coming out of a potential temperature anomaly), there will be an anomalous geostrophic wind, which changes magnitude with height away from the boundary. (Note that this relation also explains why the mean tropospheric winds are zonal, increase with height, and strongest in the mid-latitudes: there is a mean north-south gradient of potential temperature from the warm, equatorial latitudes to the cold, polar ones. From the relations above, $-\frac{d\overline{\theta}}{dy} \propto -\frac{d\overline{u}_g}{dp}$ or $+\frac{d\overline{u}_g}{dz}$.)

In figure 1, a positive potential temperature anomaly is plotted in red on a lower boundary. This anomaly decays in $x$, $y$, and $z$ away from its center. As suggested by the shrinking dashed circles directly above the surface anomaly, the anomaly decays with height until at some altitude it vanishes. I have labeled four points in the lower boundary plane containing the $\theta$ anomaly: A, B, C, and D. Let us consider the perturbation winds at each of these four points, and how they change with height.

As one heads east through point A, the gradient of potential temperature anomaly is increasing (it starts at zero some distance well west of the anomaly and increases as one heads east, passing through point A, to the center of the anomaly). From the relation expressed by equation (3), $\frac{\partial \theta'}{\partial x} > 0$ implies that an anomalous, meridional wind will decrease in value with increasing pressure (or increase in value with increasing height). Obviously, the anomalous wind owing to this temperature perturbation must decay by the same altitude as the temperature perturbation itself decays. (At this level, there is no more horizontal gradient in $\theta'$, and thus there can be no perturbation motion at this height.) With this constraint, let us now return to how the wind changes with height above point A. It is probably easiest to think of this in pressure coordinates. Given that the perturbation wind must be zero at the altitude at which all perturbations vanish, as one heads down toward the surface (increasing pressure), the value of $v_g$ must decrease. If it starts at zero, it must become negative as one heads down to the surface. Thus, at point A, there is a strong northerly wind (blowing to the south). A similar analysis may be conducted at point C, to the east of the perturbation maximum. To the east of the maximum, the value of $\theta'$ decreases as one heads east through point C until at some point, far to the east, the perturbation vanishes. Thus, $\frac{\partial \theta'}{\partial x} < 0$. Again constrained by the fact that all perturbation motions must vanish at the same altitude as $\theta'$ does, equation (3) tells us that an anomalous meridional wind will increase in value down from this pressure level to the surface. Thus, the wind is southerly at point C on the lower boundary, and decreases in intensity as one heads up in height. These winds are plotted in cyan in figure 1.

Along the line connecting point D to point B, the gradient of potential temperature perturbation is in the $y$ direction. From equation (4) the perturbation wind shears can be estimated at points B and D. Using an analysis similar to that described above, I have plotted the perturbation winds in figure 2 at and above point B (in magenta) and at and above point D (in green). Thus, it should be clear from figure 2 that the existance of a positive potential temperature anomaly along a lower boundary *implies* by thermal wind the existance of a perturbation circulation which rotates cyclonically and decreases in intensity as one heads above the surface. From equation (1) we can also learn that this temperature anomaly creates a negative geopotential perturbation (in height coordinates, a negative pressure perturbation). Thus a positive potential temperature anomaly on a lower boundary behaves exactly as a positive potential vorticity anomaly would: there is anomalous positive vorticity and anomalous negative geopotential (negative pressure).

I would encourage you to examine figures 3, 4, and 5. They feature a positive $\theta$ anomaly, but at the top boundary rather than the lower one. By examining the gradient of potential temperature moving in and out of the anomaly, and with the boundary condition that all perturbation winds vanish at some altitude below the upper surface, see if you can explain why the winds rotate anticyclonically.

Obviously temperature perturbations that are negative will produce circulations opposite in direction to the ones examined here. Try sketching some of these plots, if it would be useful.

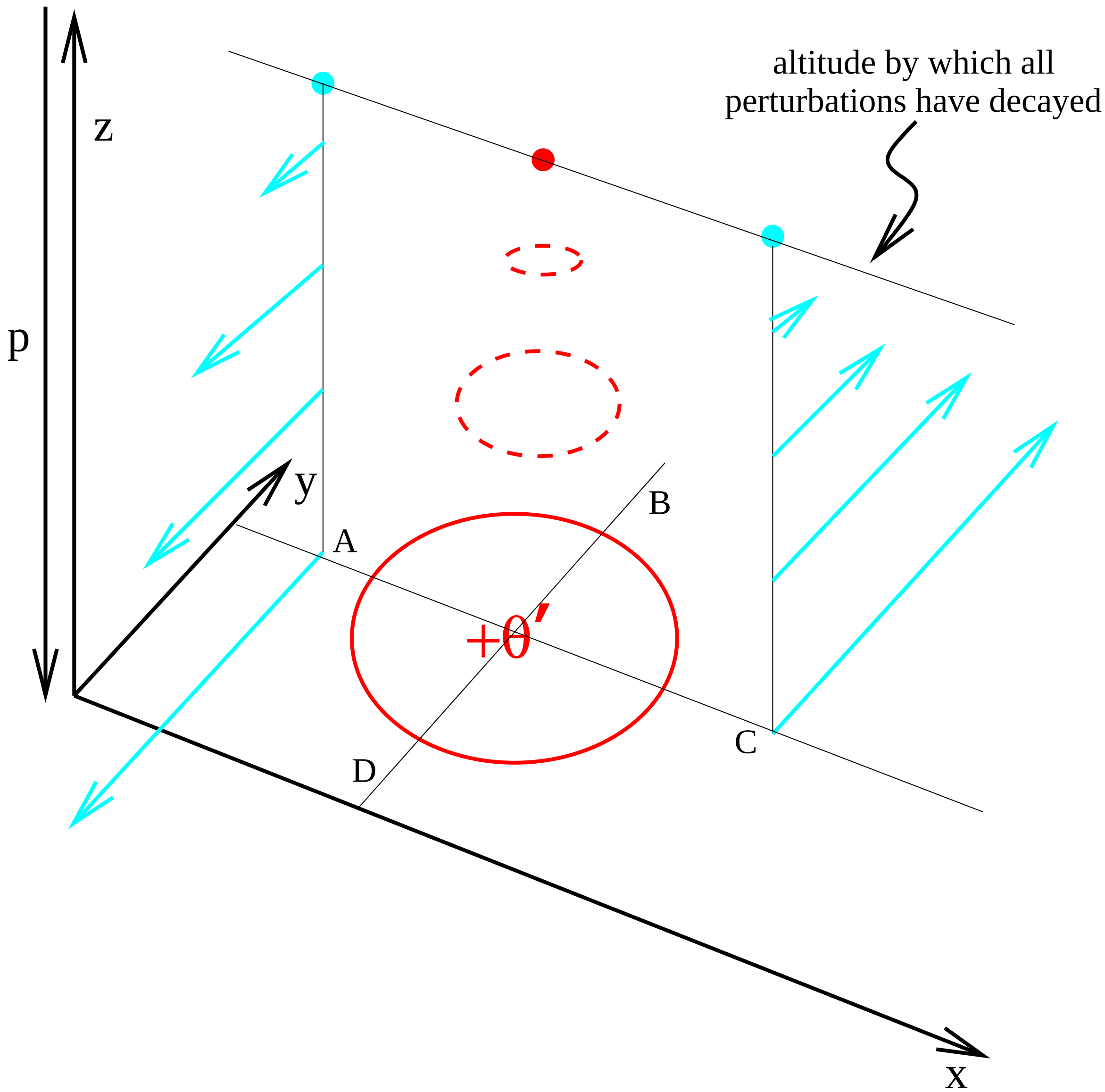


Figure 1: A positive potential temperature anomaly exists on the lower boundary (here drawn in the plane containing the $x$ and $y$ axes). The anomaly is drawn in red, and decays in intensity away from its center in $x$, $y$, and $z$. As suggested by the shrinking dashed circles above the lower boundary, the temperature anomaly decays with height until at some altitude it vanishes. Points A, B, C, and D all lie in the same lower boundary plane as the $\theta$ anomaly. The meridional wind at and above points A and C owing to the gradient of $\theta$ in the $x$ direction is plotted in cyan. The perturbation winds must vanish at the same altitude as the $\theta$ anomaly.

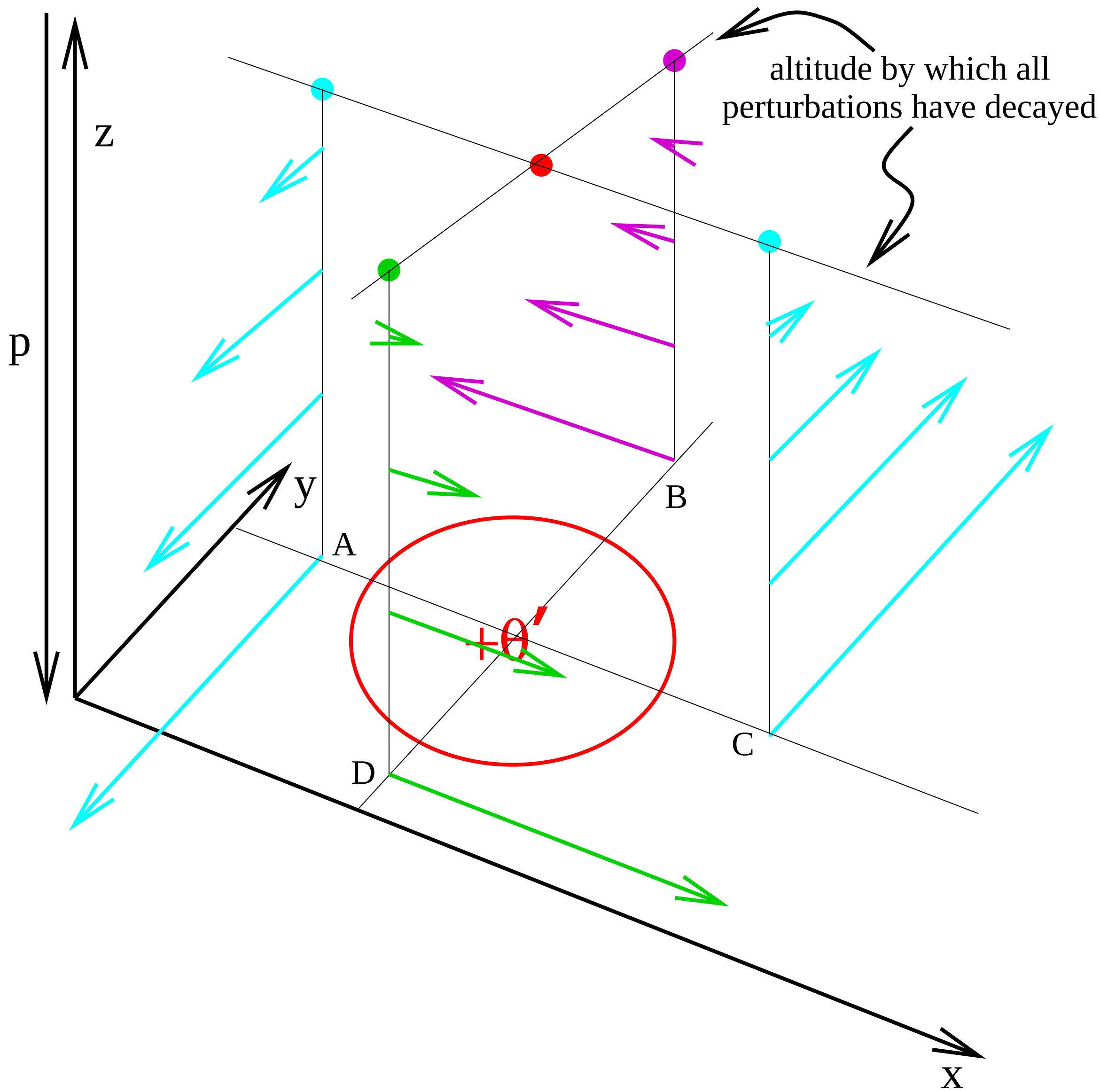


Figure 2: As in figure 1, but with perturbation winds plotted at and above points B (in magenta) and D (in green) also. At B and D, the perturbation winds are zonal as the temperature gradient is in the $y$ direction (see equation (4)).

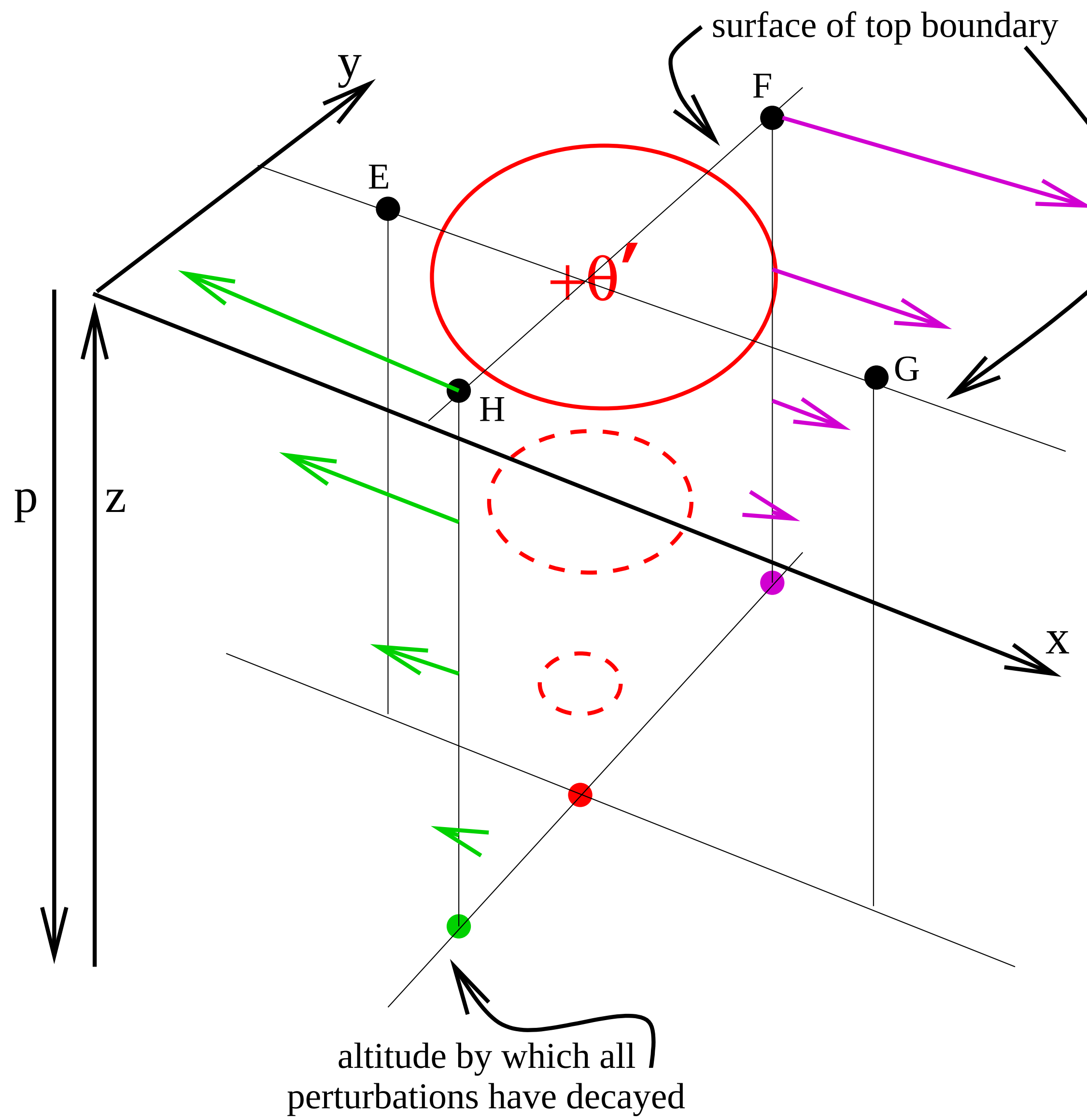


Figure 3: Points E, F, G, and H lie, along with a positive potential temperature anomaly, on the upper boundary of a fluid. The temperature anomaly is drawn in red, and dotted circles are drawn beneath the upper boundary to indicate that it decreases in intensity in $x$, $y$, and height until at some altitude beneath the top of the fluid, it vanishes entirely. The zonal winds owing to the gradient of $\theta$ in the $y$ direction are plotted at and below points F (in magenta) and H (in green). The perturbation winds must vanish at the same altitude as the $\theta$ anomaly.

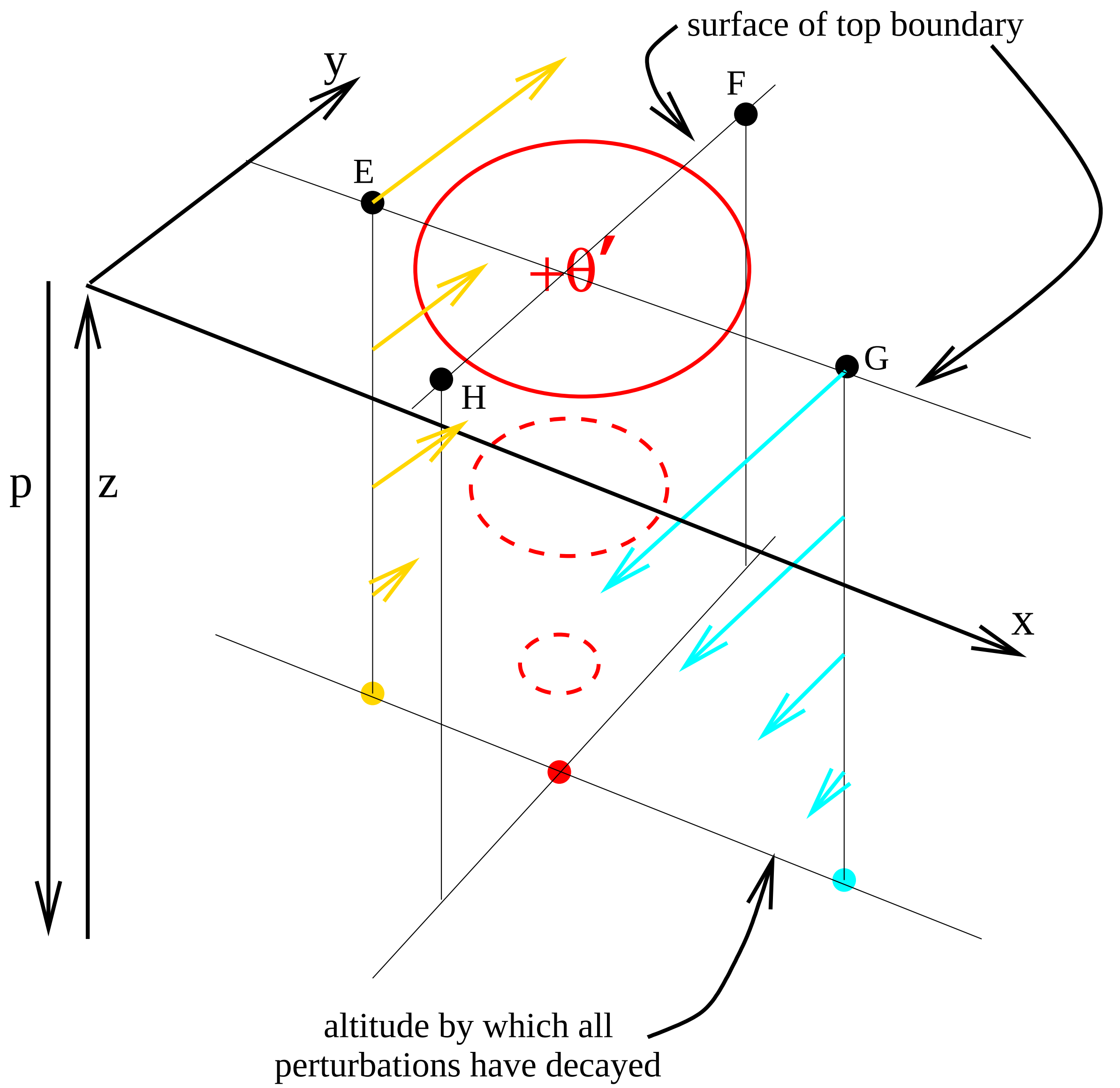


Figure 4: As in figure 3, but with perturbation winds plotted at points E (in gold) and G (in cyan). As the gradient of $\theta$ is in the $x$ direction at and below points E and G, the perturbation winds are meridional.

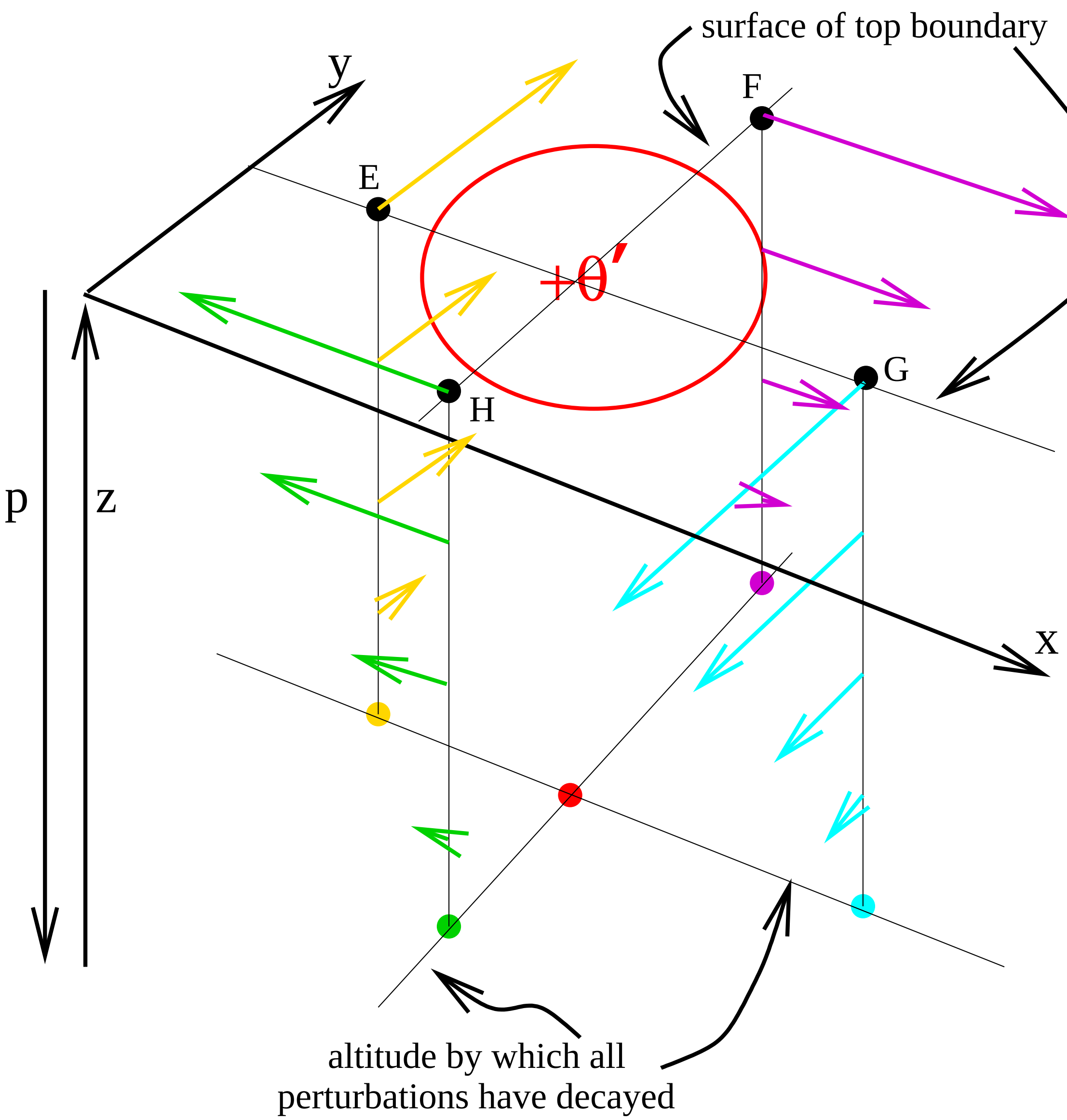


Figure 5: Figures 3 and 4 combined.