12.811

Tropical Meteorology

Problem Set 3

(Note: Doing this problem set requires notes taken in class on March 18th.)

The Held-Hou criticality condition requires zero gradients of free-tropospheric saturation entropy at and along the equator. On the other hand, large east-west gradients in ocean temperature are observed along the equator in the Pacific Ocean. The object of this exercise in to estimate the large-scale vertical motion and convective upward mass flux along the equator when such an ocean temperature gradient exists.

The ocean temperature gradient affects the atmosphere through its effect on surface fluxes. We will approximate the surface entropy flux, F_s by a bulk aerodynamic formula:

$$F_s = C_k \, |\mathbf{V}| \left(s_0 * - s_b \right), \tag{1}$$

where C_k is a nondimensional coefficient, $|\mathbf{V}|$ is a near-surface wind speed, $s_0 *$ is the saturation entropy of the ocean surface (a function of sea surface temperature and pressure only), and s_b is the entropy of the subcloud layer.

If we assume convective neutrality, then $s_b = s^*$, where the latter is the saturation entropy of the free troposphere (which is also constant in altitude, according to convective neutrality). Held-Hou criticality also demands that $s*$ not vary with longitude along the equator.

To make life simple, let's assume that the ocean temperature varies along the equator in just such a way that $s_0 *$ varies linearly with distance:

$$s_0 * = s_1 - \frac{x}{L} \Delta s, \tag{2}$$

where x is the distance eastward along the equator, defined such that $x = 0$ in the middle of the basin, L is the half-width of the basin, and Δs is half the total difference of $s_0 *$ across the basin.

1. Write down expressions for the large-scale vertical velocity and upward convective mass flux as functions of x. Assuming that the radiative cooling velocity, $\dot{Q}_{cool}/S$, the wind speed $|\mathbf{V}|$, the middle troposphere entropy, s_m, and the constants ε_p and γ are all constants, write down an expression for s^* that guarantees that the large-scale vertical velocity, averaged over the whole basin, vanishes.

2. Using the expression for s^* derived in 1) above, re-write the expressions for large-scale vertical velocity and upward convective mass flux as functions of x. Suppose that $\varepsilon_p C_k |\mathbf{V}| = \dot{Q}_{cool}/S$ and that $\gamma = 0.1; \quad \varepsilon_p = 0.8$. How large does $\Delta s / (s_1 - s_m)$ have to be to invalidate the solution in at least part of the domain? Explain your reasoning.

3. Suppose that the condition derived in 2) is indeed violated over part of the domain. For that part of the domain, write down expressions for the large-scale vertical velocity and for the subcloud layer entropy, s_b, which you can now assume is decoupled from s^*. Determine a condition for s_b to be less than s^*.