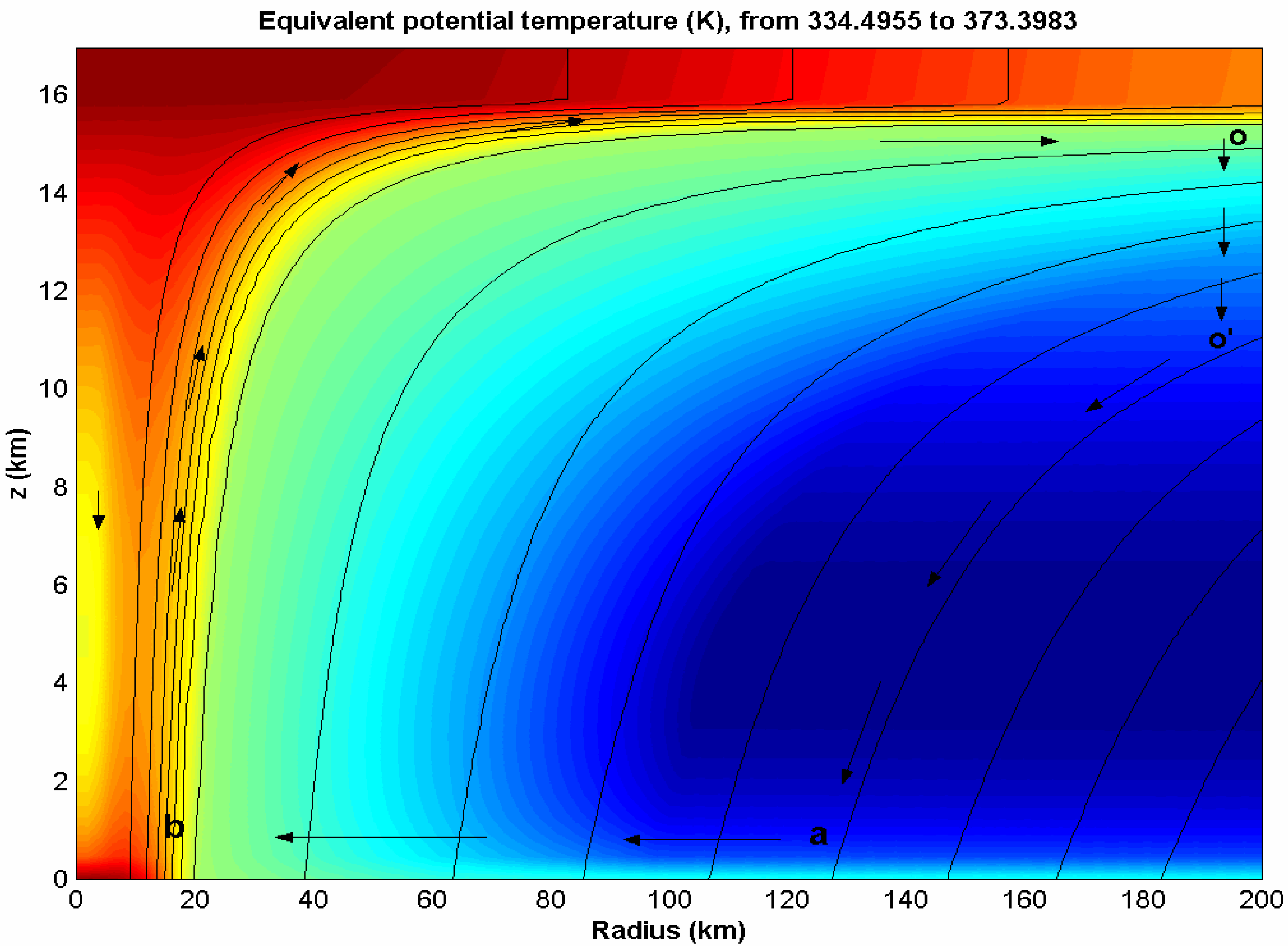
Tropical Cyclones: Steady State Physics

Energy Production



Carnot Theorem: Maximum efficiency results from a particular energy cycle:

- Isothermal expansion
- Adiabatic expansion
- Isothermal compression
- Adiabatic compression

Note: Last leg is not adiabatic in hurricane: Air cools radiatively. But since environmental temperature profile is moist adiabatic, the amount of radiative cooling is the same as if air were saturated and descending moist adiabatically.

Maximum rate of working:

$$W = \frac{T_s - T_o}{T_s} \dot{Q}$$

Total rate of heat input to hurricane:

$$\dot{Q} = 2\pi \int_0^{r_0} \rho \left[C_k \,|\, \mathbf{V} \,|\, \left(k_0^* - k \right) + C_D \,|\, \mathbf{V} \,|^3 \right] r dr$$

Surface enthalpy flux

Dissipative heating

In steady state, Work is used to balance frictional dissipation:

$$W = 2\pi \int_0^{r_0} \rho \left[C_D \,|\, \mathbf{V} \,|^3 \right] r dr$$

Plug into Carnot equation:

$$\int_0^{r_0} \rho\left[C_D |\mathbf{V}|^3\right] r dr = \frac{T_s - T_o}{T_o} \int_0^{r_0} \rho\left[C_k |\mathbf{V}| \left(k_0^* - k\right)\right] r dr$$

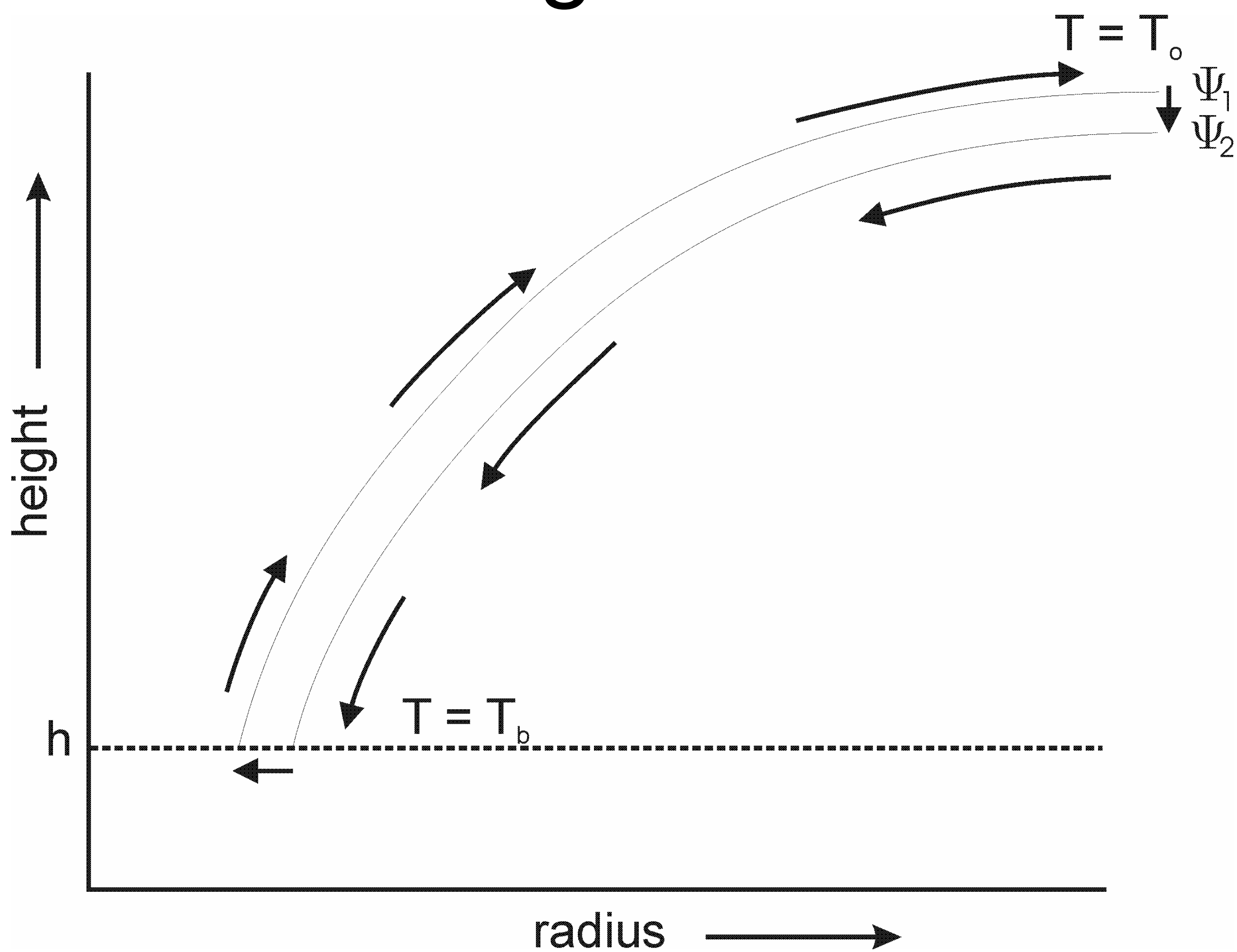
If integrals dominated by values of integrands near radius of maximum winds,

$$\rightarrow \quad |V_{\max}|^2 \cong \frac{C_k}{C_D} \frac{T_s - T_o}{T_o} \left(k_0{}^* - k\right)$$

Problems with Energy Bound:

- Implicit assumption that all irreversible entropy production is by dissipation of kinetic energy. But outside of eyewall, cumuli moisten environment....accounting for almost all entropy production there
- Approximation of integrals dominated by high wind region is crude

Local energy balance in eyewall region:



Definition of streamfunction, ψ:

$$\rho r w = \frac{\partial \psi}{\partial r}, \qquad \rho r u = -\frac{\partial \psi}{\partial z}$$

Flow parallel to surfaces of constant ψ, satisfies mass continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}(\rho r u) + \frac{1}{r}\frac{\partial}{\partial z}(\rho r w) = 0$$

Variables conserved (or else constant along streamlines) above PBL, where flow is considered reversible, adiabatic and axisymmetric:

Energy:

$$E = c_p T + L_v q + gz + \frac{1}{2} |\mathbf{V}|^2$$

Entropy:

$$s^* = c_p \ln T - R_d \ln p + \frac{L_v q^*}{T}$$

Angular Momentum:

$$M = rV + \frac{1}{2} f r^2$$

First definition of *s*:*

$$T ds^* = c_p dT + L_v dq^* - \alpha dp \qquad (1)$$

Steady flow:

$$\alpha dp = \alpha \frac{\partial p}{\partial r} dr + \alpha \frac{\partial p}{\partial z} dz$$

Substitute from momentum equations:

$$\alpha dp = -dz\left[g + \mathbf{V} \bullet \nabla w\right] + dr\left[\frac{V^2}{r} + fV - \mathbf{V} \bullet \nabla u\right] \qquad (2)$$

Identity:

$$\left(\mathbf{V}\bullet\nabla w\right)dz+\left(\mathbf{V}\bullet\nabla u\right)dr=\frac{1}{2}d\left(u^2+w^2\right)+\frac{1}{\rho r}\varsigma d\psi, \quad (3)$$

where $\varsigma\equiv\dfrac{\partial u}{\partial z}-\dfrac{\partial w}{\partial r}$ azimuthal vorticity

Substituting (3) into (2) and the result into (1) gives:

$$Tds^*=dE-VdV-\left(\frac{V^2}{r}+fV\right)dr+\frac{1}{\rho r}\varsigma d\psi \qquad (4)$$

One more identity:

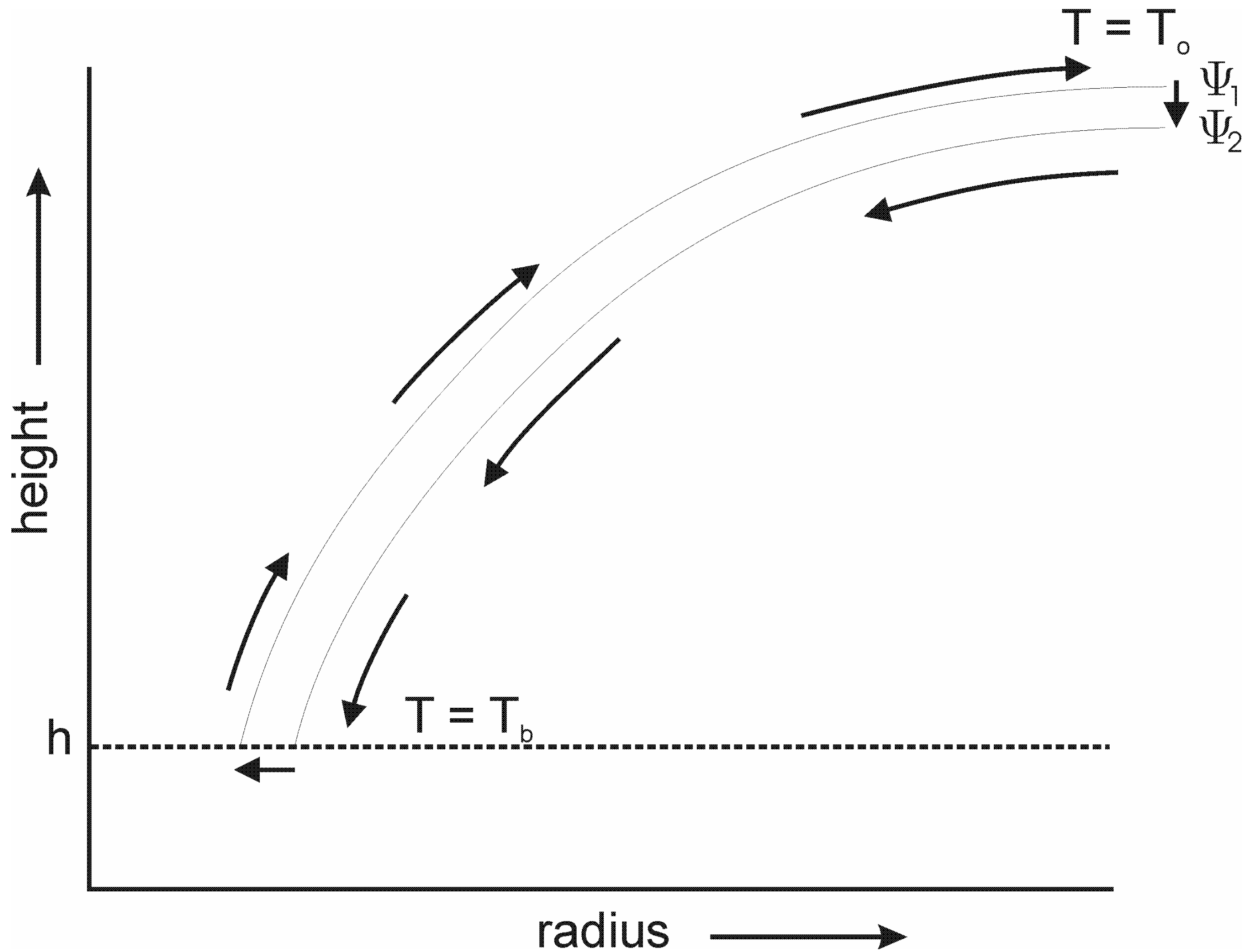
$$VdV + \left(\frac{V^2}{r} + fV \right) dr = \left(\frac{M}{r^2} - \frac{1}{2} f \right) dM$$

Substitute into (4):

$$Tds^{*} + \frac{M}{r^2} dM - \frac{1}{\rho r} \xi d\psi = d\left[E + \frac{1}{2} fM \right] \qquad (5)$$

Note that third term on left is very small: Ignore

Integrate (5) around closed circuit:



Right side vanishes; contribution to left only from end points

$$\left(T_b - T_o\right) ds^* + \left(\frac{1}{r_b^2} - \frac{1}{r_o^2}\right) M dM = 0$$

$$\rightarrow \frac{1}{r_b^2} = \frac{1}{r_o^2} - \left(T_b - T_o\right) \frac{ds^*}{MdM} \qquad (6)$$

Mature storm: $r_o >> r_b$:

$$\rightarrow \frac{M}{r_b^2} \cong -\left(T_b - T_o\right) \frac{ds^*}{dM} \qquad (7)$$

In inner core, $V >> fr$

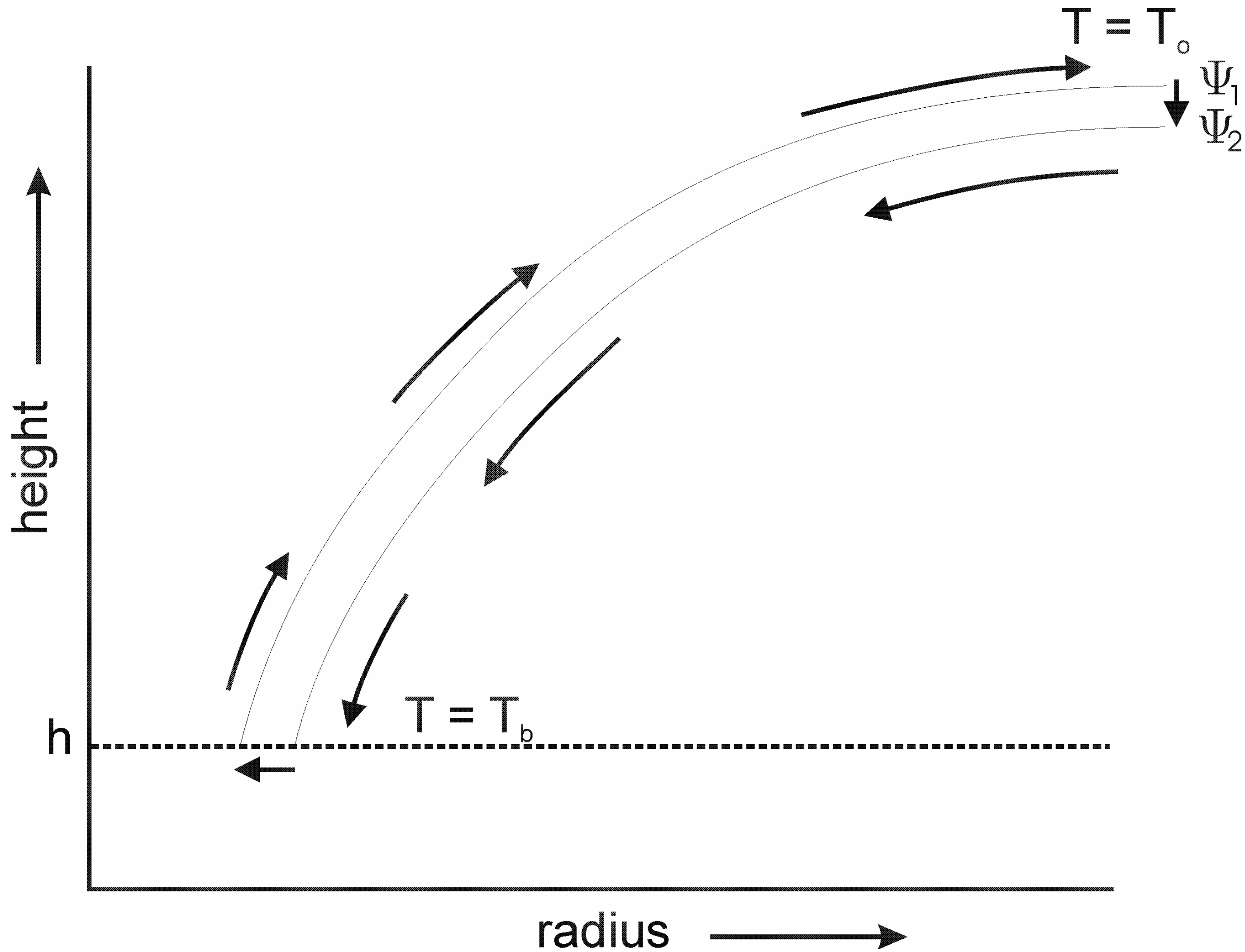
$$\rightarrow M \cong rV$$

$$V_b \cong -r_b \left(T_b - T_o \right) \frac{ds^*}{dM} \qquad (8)$$

Convective criticality: $s^* = s_b$

$$\rightarrow V_b \cong -r_b \left(T_b - T_o \right) \frac{ds_b}{dM} \qquad (9)$$

ds_b/dM determined by boundary layer processes:



Put (7) in differential form:

$$\left(T_b - T_o\right)\frac{ds}{dt} + \frac{M}{r^2}\frac{dM}{dt} = 0, \qquad (10)$$

Integrate entropy equation through depth of boundary layer:

$$h\frac{d\overline{s}}{dt} = \frac{1}{T_s}\left[C_k\,|\mathbf{V}|\left(k_s^{\,*} - k\right) + C_D\,|\mathbf{V}|^3 + F_b\right], \qquad (11)$$

where F_b is the enthalpy flux through PBL top. Integrate angular momentum equation through depth of boundary layer:

$$h\frac{d\overline{M}}{dt} = -C_D r\,|\mathbf{V}|\,V \qquad (12)$$

Substitute (11) and (12) into (10) and set F_b to 0:

$$\rightarrow \quad |V|^2 = \frac{C_k}{C_D}\frac{T_s - T_o}{T_o}\left(k_0^{\ *} - k\right) \qquad (13)$$

Same answer as from Carnot cycle. This is still not a closed expression, since we have not determined the boundary layer enthalpy, *k*. We can do this using boundary layer quasi-equilibrium as follows. First, use *moist static energy*, *h* instead of *k*:

$$|V|^2 = \frac{C_k}{C_D}\frac{T_s - T_o}{T_o}\left(h_0^{\ *} - h\right) \qquad (14)$$

$$h \equiv k + gz = c_p T + L_v q + gz$$

Convective neutrality: $$h_b = h^*$$

First law of thermo:

$$dh_b^* = T_b ds^* + R_d T_b d\ln p, \qquad (15)$$

Go back to equation (10):

$$\left(T_b - T_o\right)\frac{ds}{dt} + \frac{M}{r^2}\frac{dM}{dt} = 0$$

Use definition of M and gradient wind balance:

$$R_d T \frac{\partial \ln p}{\partial r} = \frac{V^2}{r} + fV$$

$$\rightarrow \left(T_b - T_o\right) ds^* = -\left[d\left(\frac{1}{2}V^2 + \frac{1}{2} frV\right) + \frac{1}{2} f^2 r dr + \left(\frac{V^2}{r} + fv\right) dr \right]$$

$$= -d\left[\frac{1}{2}V^2 + \frac{1}{2} frV + \frac{1}{4} f^2 r^2 + R_d T \ln p\right].$$

Substitute into (15):

$$dh_b^* = -\frac{T_b}{T_b - T_o} d\left[\frac{1}{2}V^2 + \frac{1}{2} frV + \frac{1}{4} f^2 r^2 + R_d T_o \ln p\right]$$

Define an outer radius, r_a, where $V=0$, $p=p_o$. Take difference between this and radius of maximum winds:

$$h_b^* = h_a^* - \frac{T_b}{T_b - T_o}\left[\frac{1}{2}\left(V_{max}^2 + fr_m V_{max}\right) + R_d T_o \ln\left({}^{p_m}\!/_{p_a}\right) - \frac{1}{4} f^2 r_a^2\right]$$

Relate p_m to V_{max} using gradient wind equation. But simpler to use an empirical relation:

$$R_d T_s \ln \frac{p_m}{p_a} \cong -b V_{max}^2$$

Also neglect fr_m in comparison to $V_{max.}$ and neglect difference between T_s and T_b:

Substitute into (14):

$$V_{max}^2 \cong \frac{C_k}{C_D}\left[\frac{\frac{T_s - T_o}{T_o}\left(h_s^* - h_a^*\right) - \frac{1}{4}\frac{T_s}{T_o} f^2 r_a^2}{1 - \frac{C_k}{C_D}\left(\frac{1}{2}\frac{T_s}{T_o} - b\right)}\right].$$

Absolute upper bound on storm size:

$$r_{o\,max}^2 = 4\frac{T_s - T_o}{T_s}\frac{\left(h_s^* - h_a\right)}{f^2}$$

$$r_{o\,max} \approx 1000\ km$$

For $r_o << r_{o\ max}$ neglect last term in numerator.

$$V_{max}^2 \cong \frac{C_k}{C_D}\frac{T_s - T_o}{T_o}\left(h_s^* - h_a^*\right) \qquad (16)$$

$$h_s^* - h_a^* = h_s^* - h_{ab} = c_p\left(T_s - T_b\right) + L_v\left(q_s^* - q_{ab}\right)$$

$$\cong L_v q_{sa}^*\left(1 - \mathscr{H}_a\right)$$

$$= L_v \varepsilon \frac{e^*(T_s)}{p}\left(1 - \mathscr{H}_a\right)$$

Since *p* depends on *V*, this makes (16) an implicit equation for *V*. But write expression for wind force instead:

$$\rho V_{max}^2 = \frac{p}{R_d T_s} V_{max}^2 = \frac{C_k}{C_D} \frac{T_s - T_o}{T_o} e^*(T_s) \frac{L_v}{R_v T_s}\left(1 - \mathscr{H}_a\right)$$

Otherwise, use empirical relationship between *V* and *p*:

$$R_d T_s \ln {p}/{p_a} \cong -bV^2$$

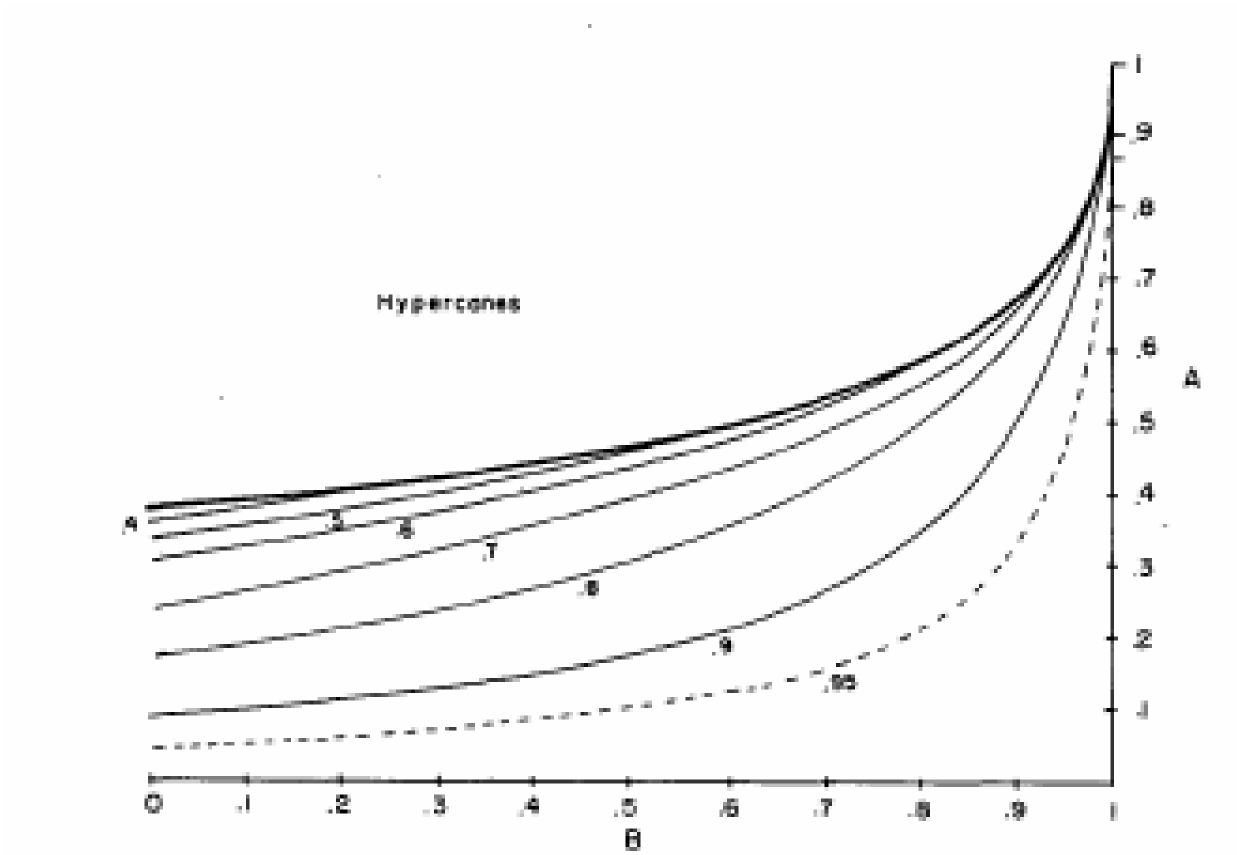
$$p = p_a e^{-bV^2/R_d T_s}$$

$$V_{max}^2 \cong \frac{C_k}{C_D}\frac{T_s - T_o}{T_o}\left(h_s^* - h_a^*\right) = \frac{C_k}{C_D}\frac{T_s - T_o}{T_o} L_v \varepsilon \frac{e^*(T_s)}{p}\left(1 - \mathcal{H}_a\right)$$

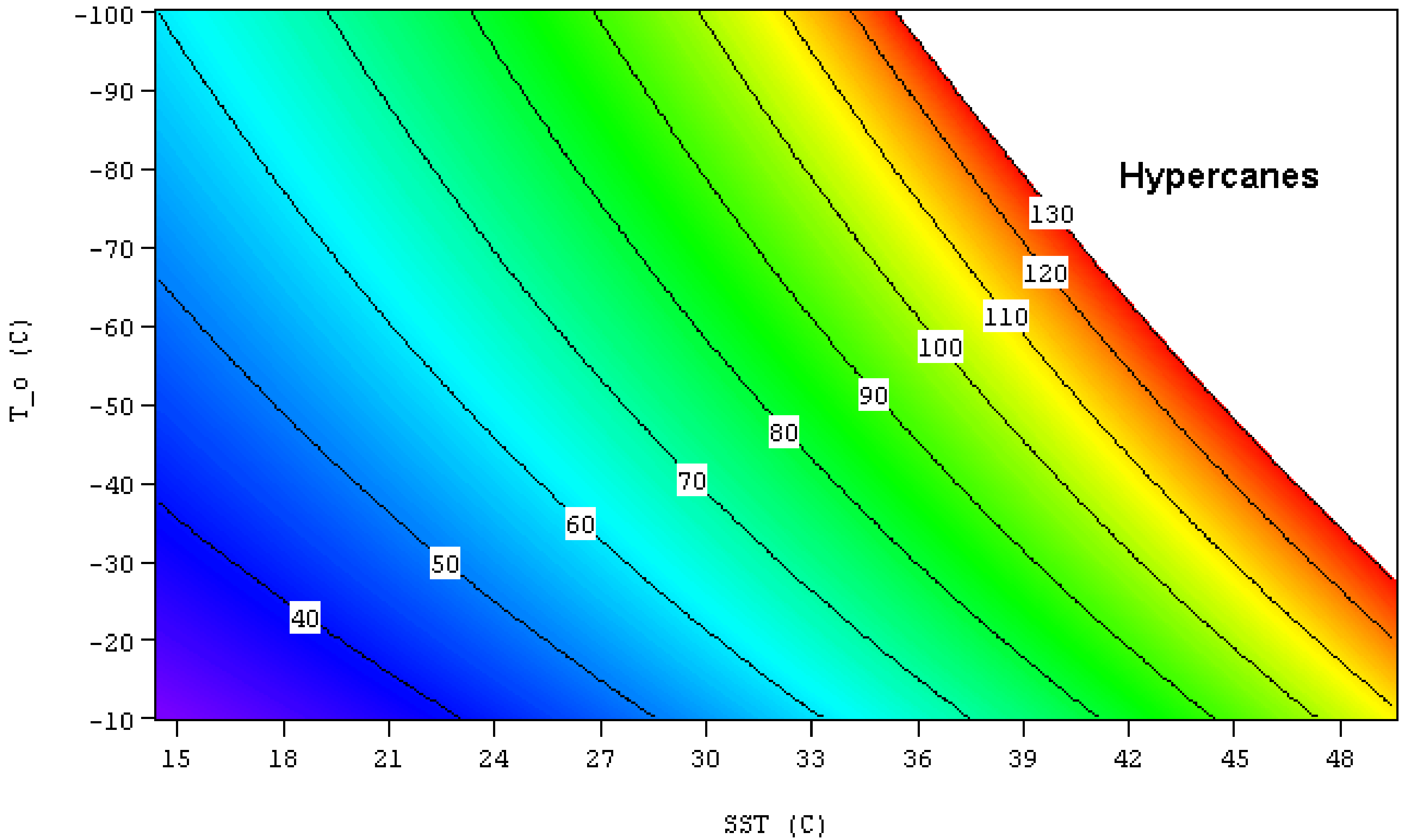
$$\rightarrow p_a e^{-bV_{max}^2/R_d T_s} V_{max}^2 \cong \frac{C_k}{C_D}\frac{T_s - T_o}{T_o} L_v \varepsilon e^*(T_s)\left(1 - \mathcal{H}_a\right)$$

Taking natural log of this, the result can be written in the form:

$$AV_{max}^2 - \ln\left(V_{max}^2\right) - B = 0$$

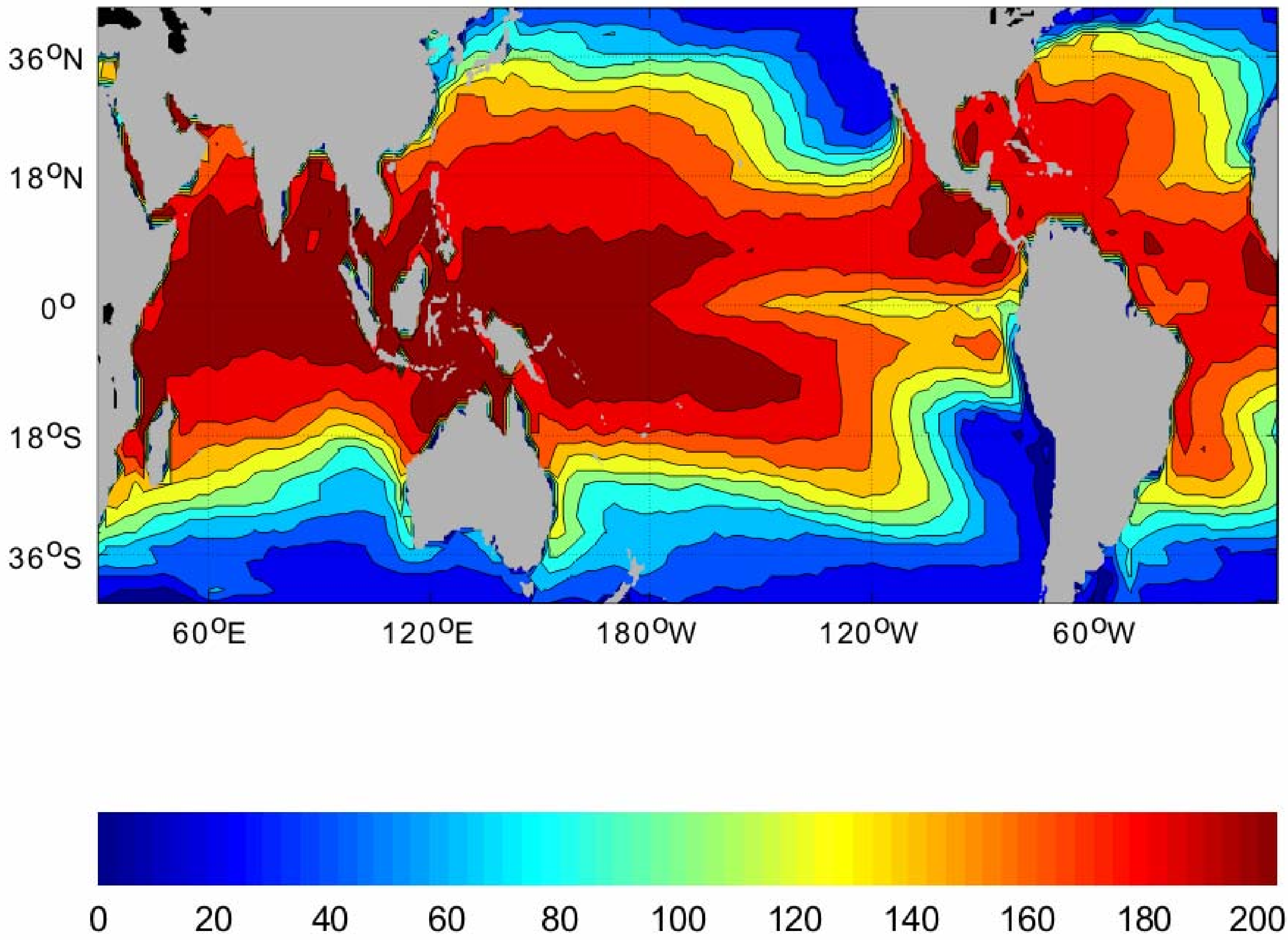


Maximum Wind Speed (m/s)

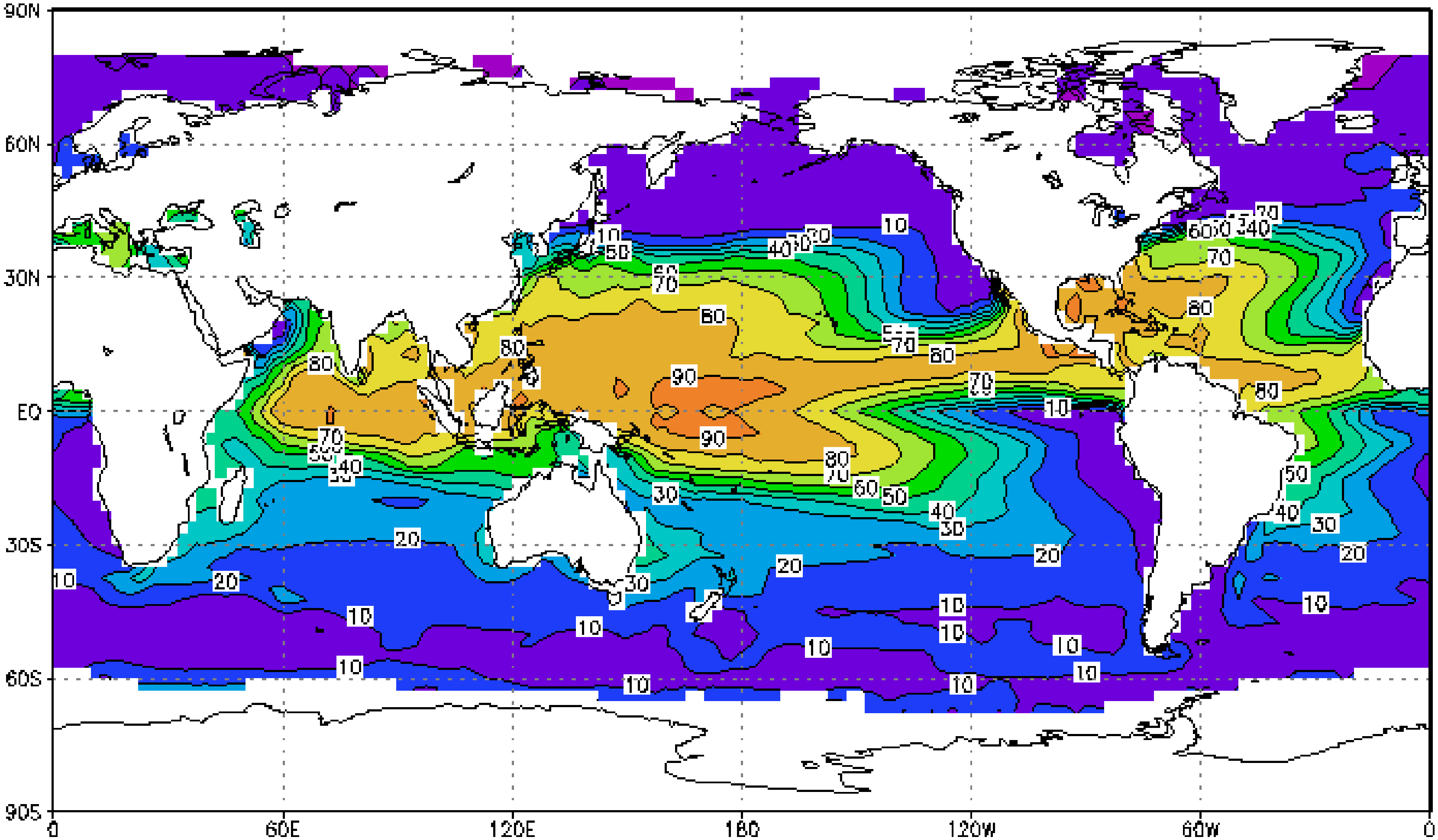


$\mathcal{H} = 0.75 \quad C_k/C_D = 1.2$

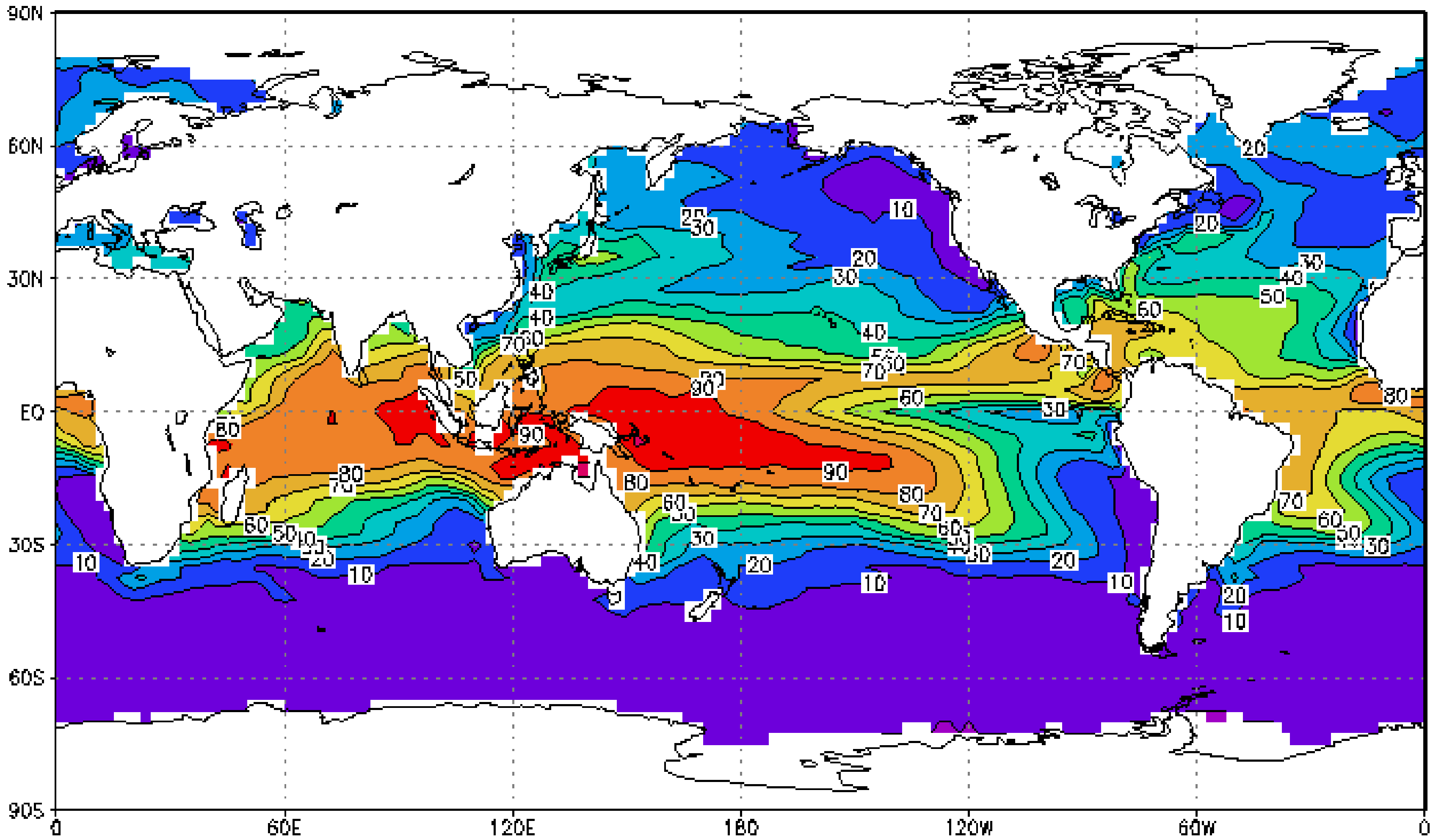
Maximum Annual Potential Intensity (MPH)

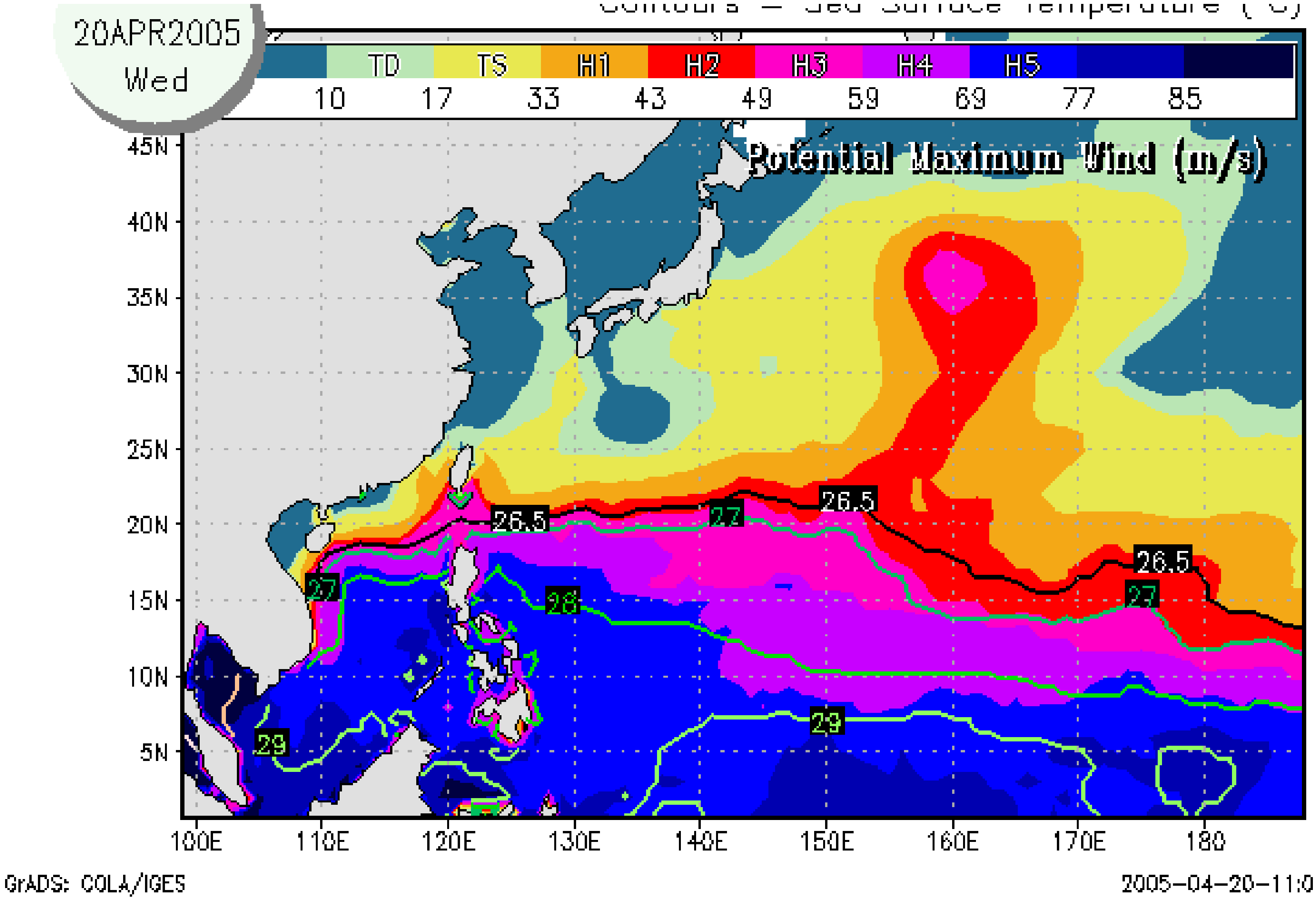


AUGUST MEAN



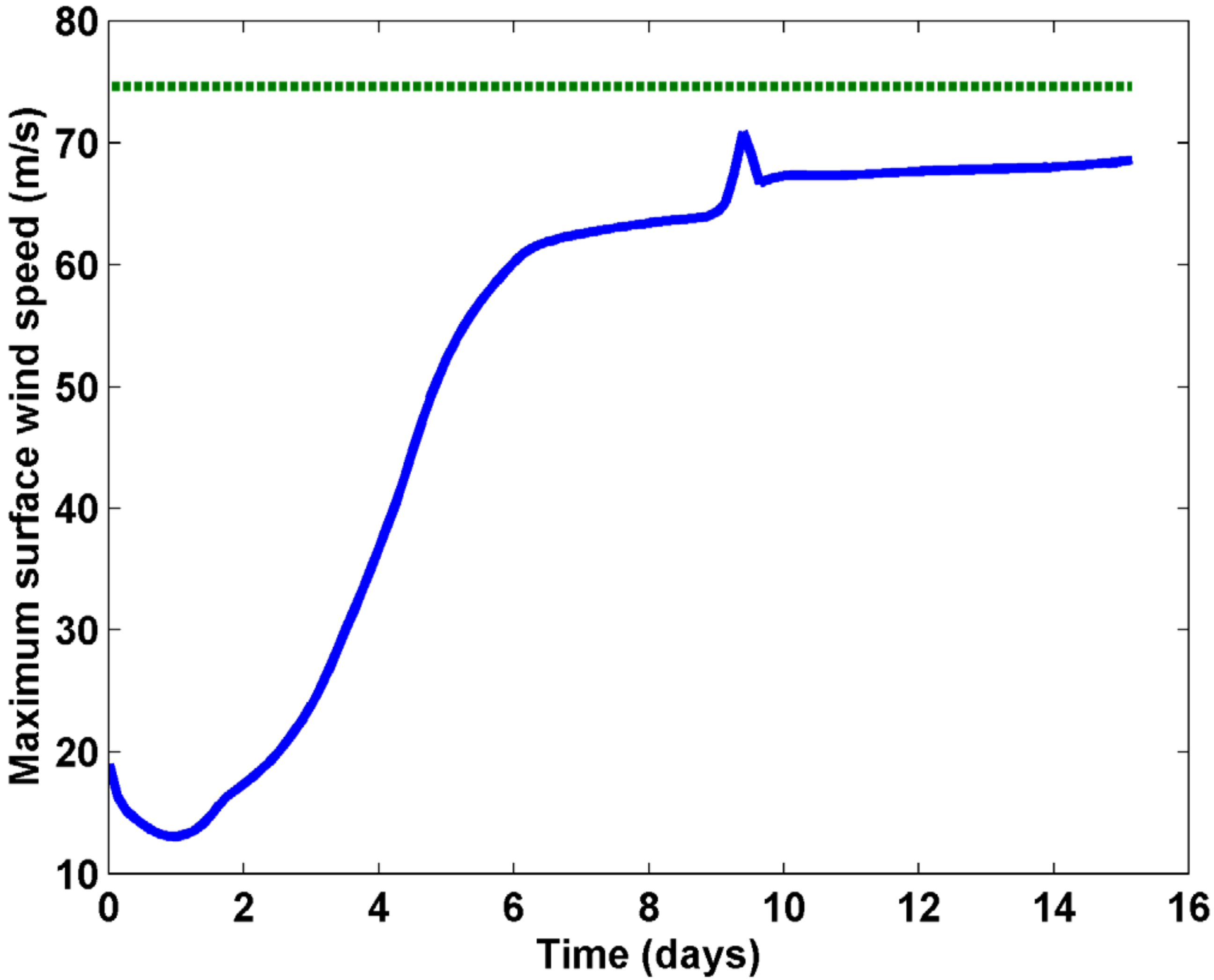
JANUARY MEAN



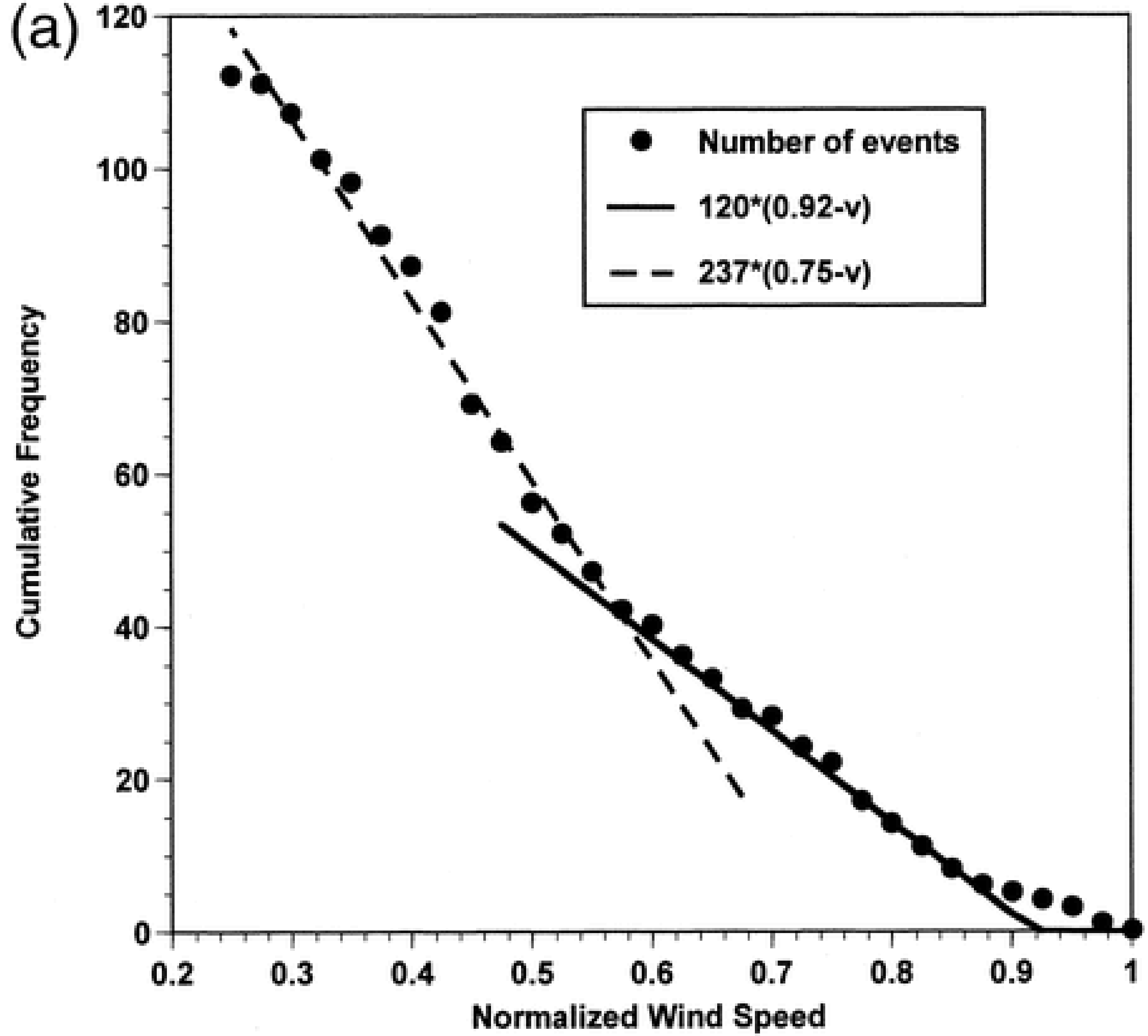


20APR2005
Wed
TD
TS
H1
H2
H3
H4
H5
10
17
33
43
49
59
69
77
85
Potential Maximum Wind (m/s)
45N
40N
35N
30N
25N
20N
15N
10N
5N
26.5
27
28
29
100E
110E
120E
130E
140E
150E
160E
170E
180
GrADS: COLA/IGES
2005-04-20-11:01

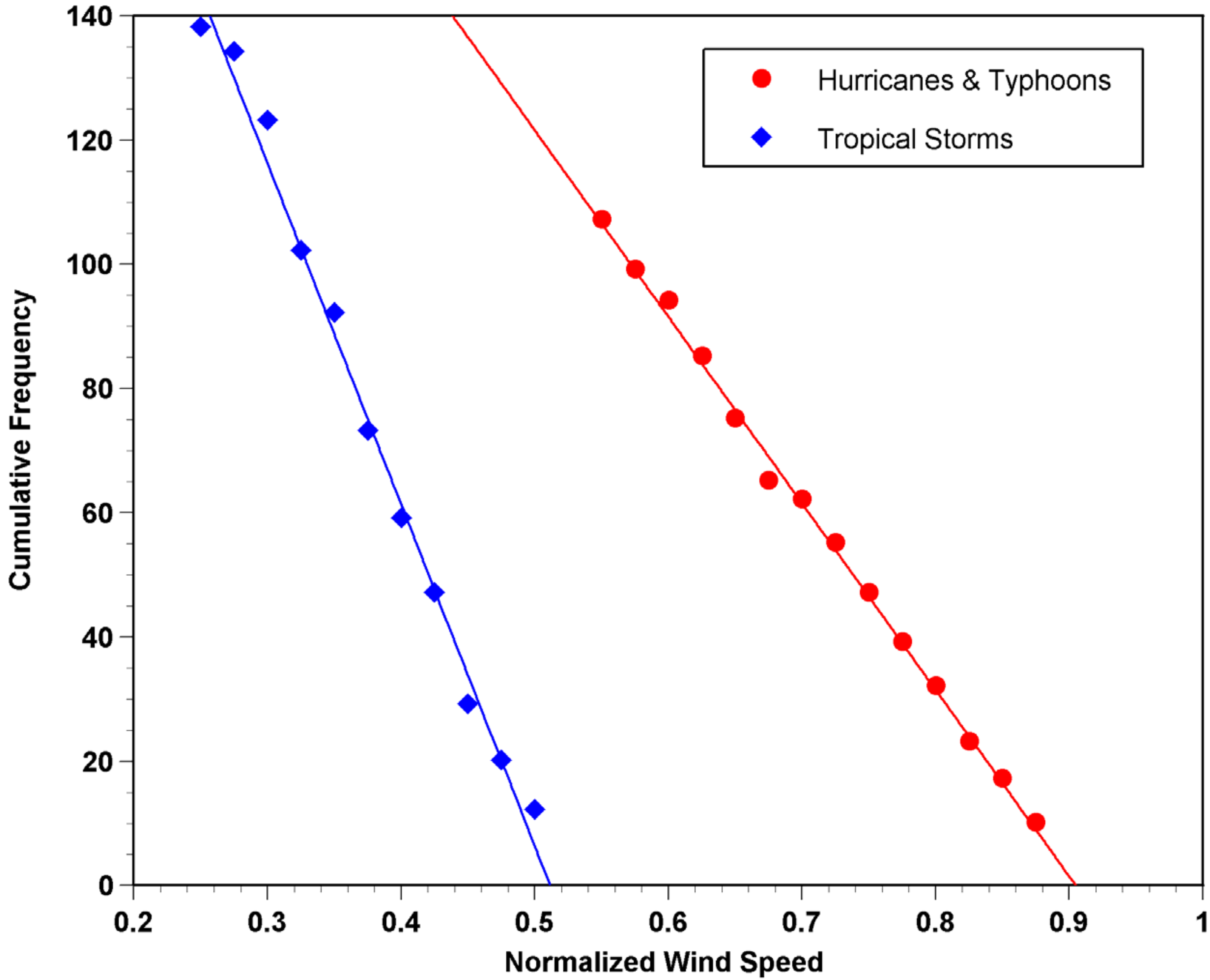
Numerical simulations



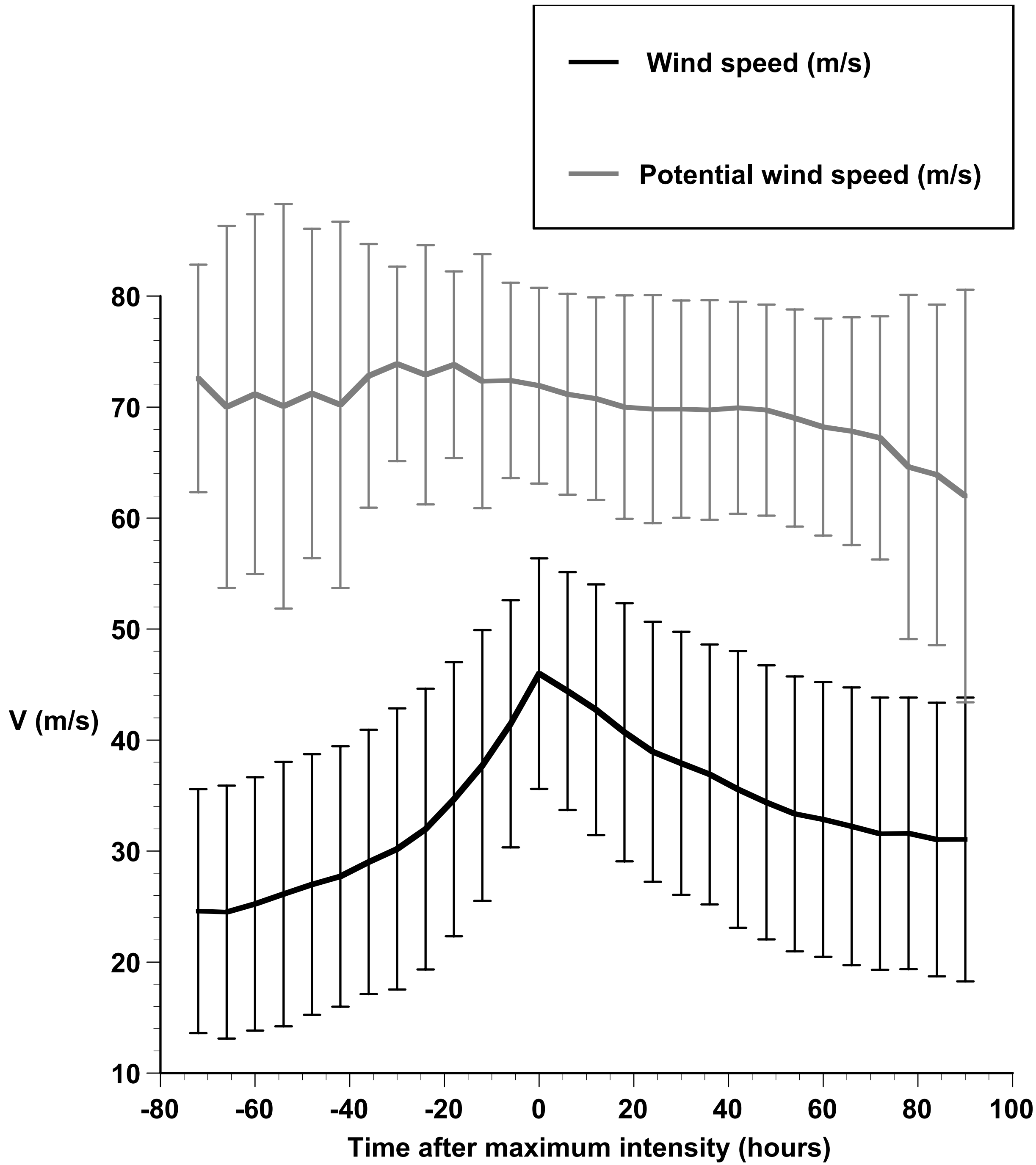
Relationship between potential intensity (PI) and intensity of real tropical cyclones



(a)
120
100
80
60
40
20
0
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Cumulative Frequency
Normalized Wind Speed
Number of events
120*(0.92-v)
237*(0.75-v)

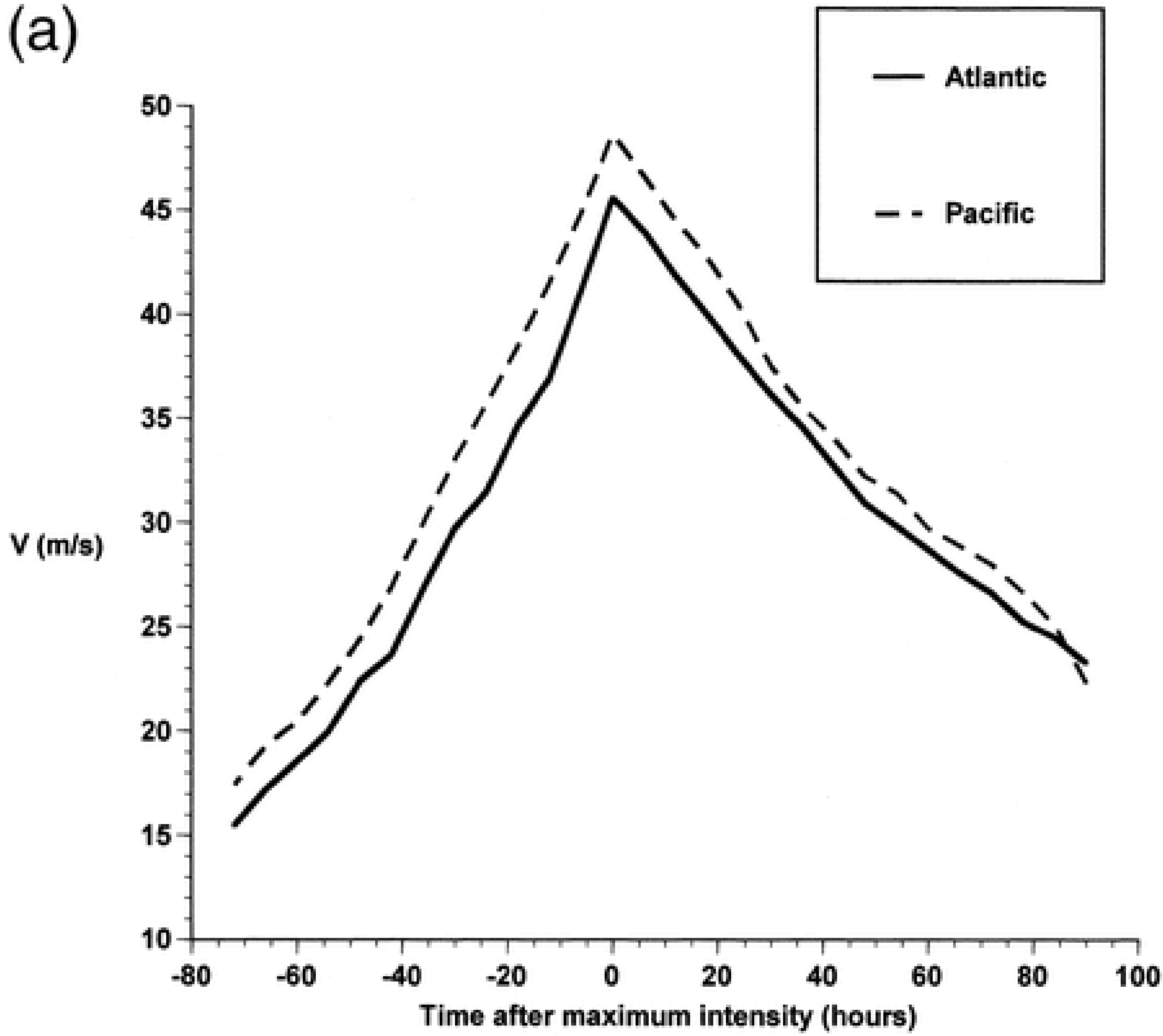


140
120
100
80
60
40
20
0
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Cumulative Frequency
Normalized Wind Speed
Hurricanes & Typhoons
Tropical Storms

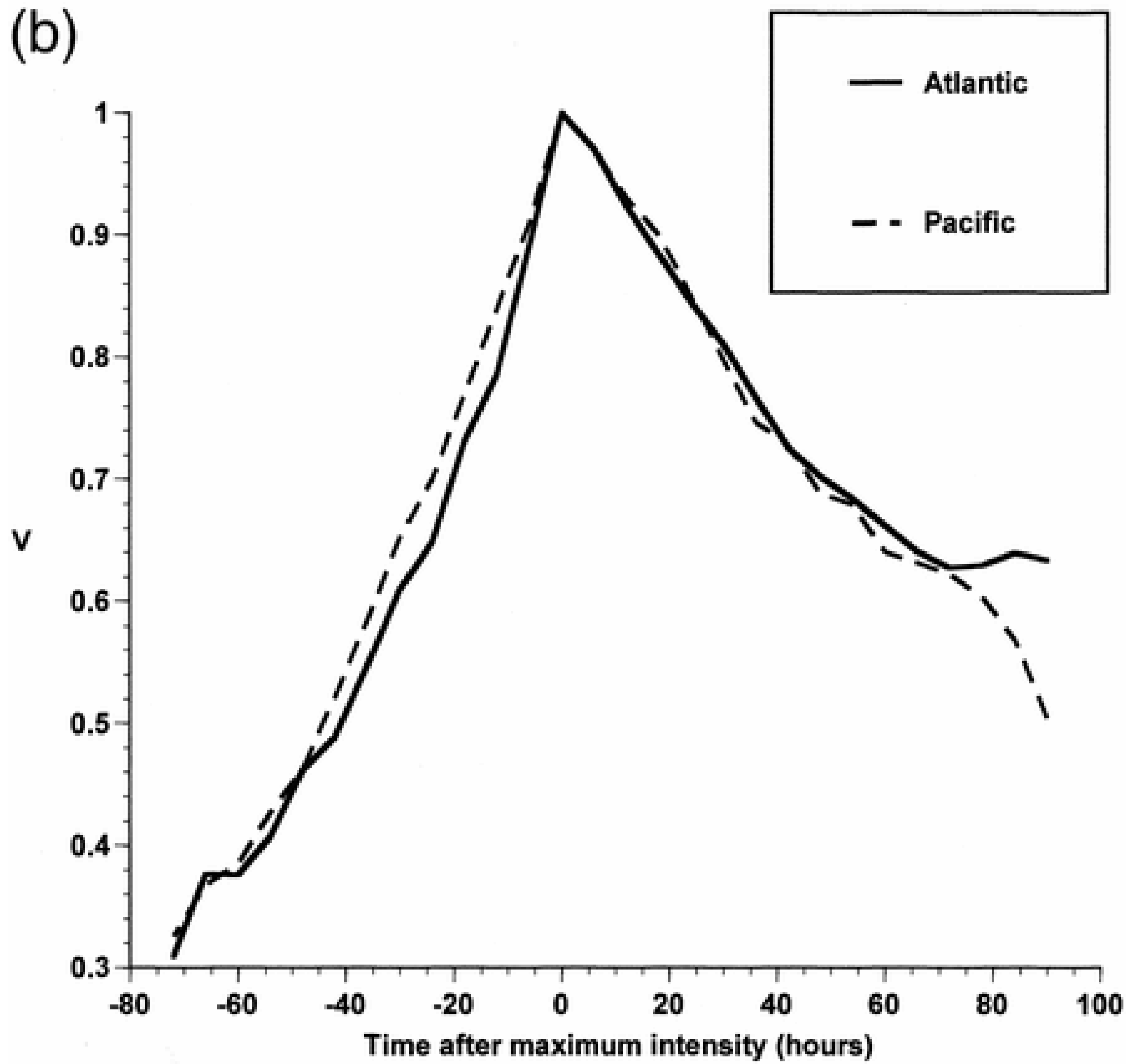


Wind speed (m/s)
Potential wind speed (m/s)
V (m/s)
80
70
60
50
40
30
20
10
-80
-60
-40
-20
0
20
40
60
80
100
Time after maximum intensity (hours)

Evolution with respect to time of maximum intensity

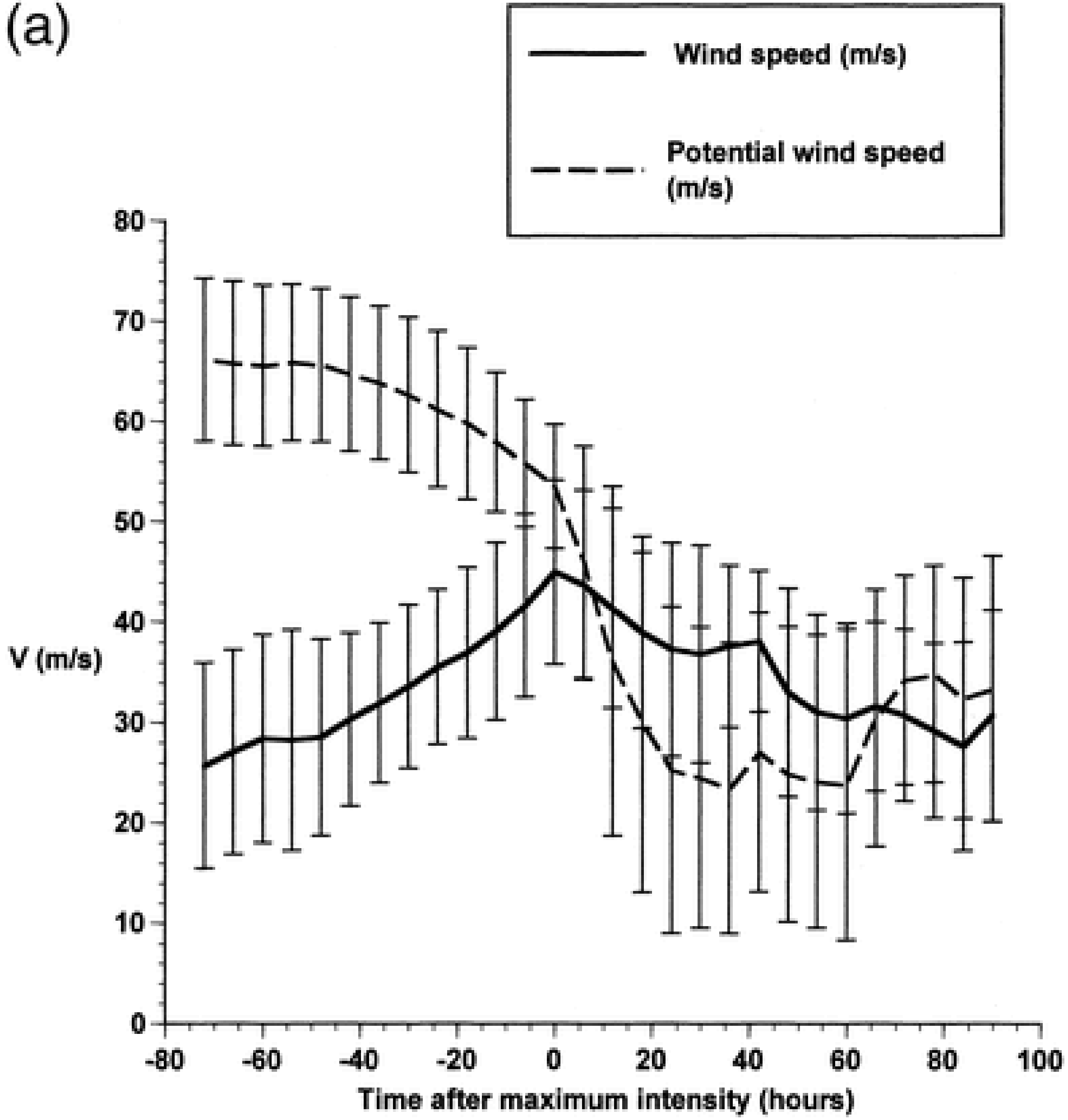


Evolution with respect to time of maximum intensity, normalized by peak wind

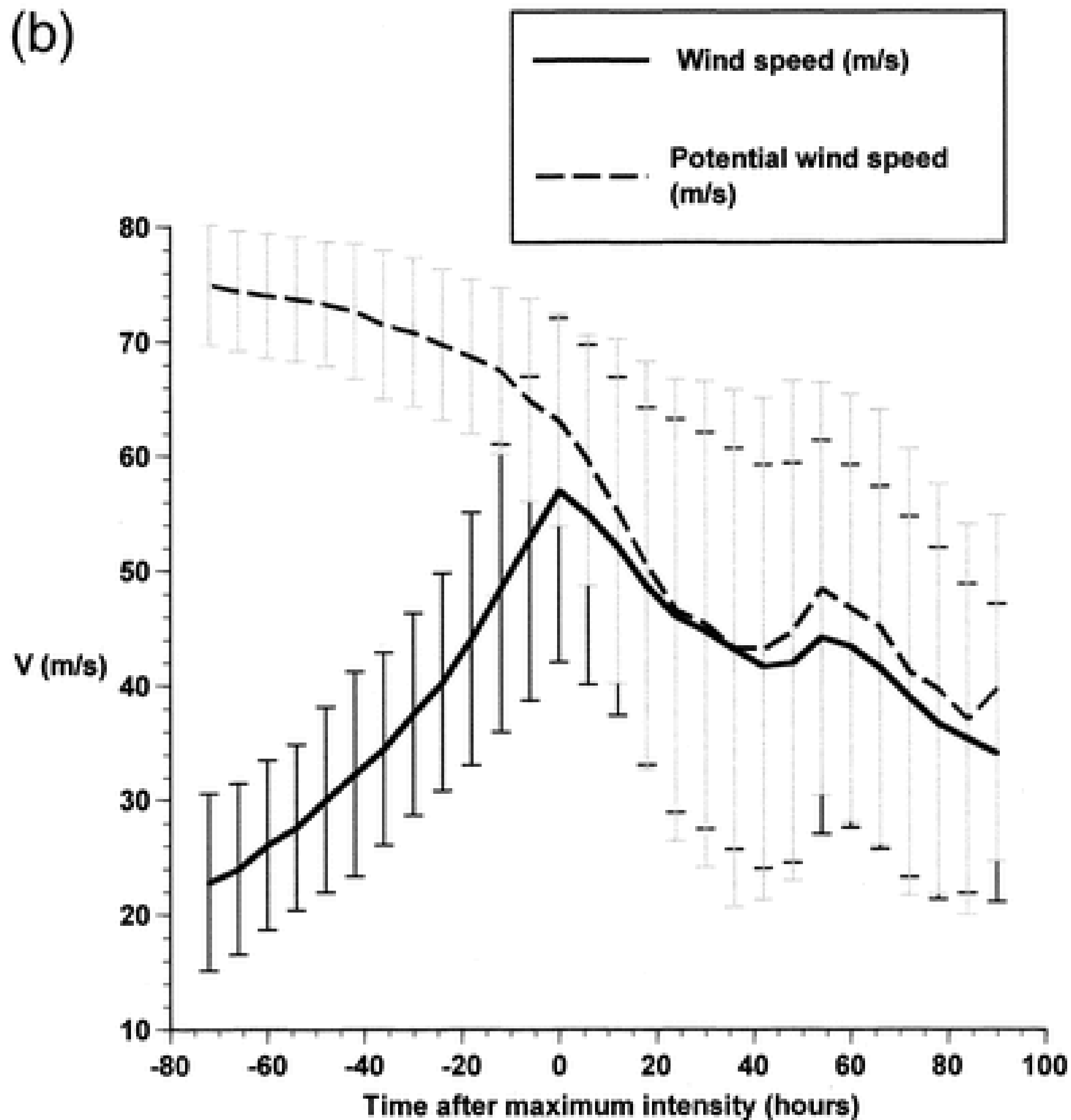


Evolution curve of Atlantic storms whose lifetime maximum intensity is limited by declining potential intensity, but not by landfall

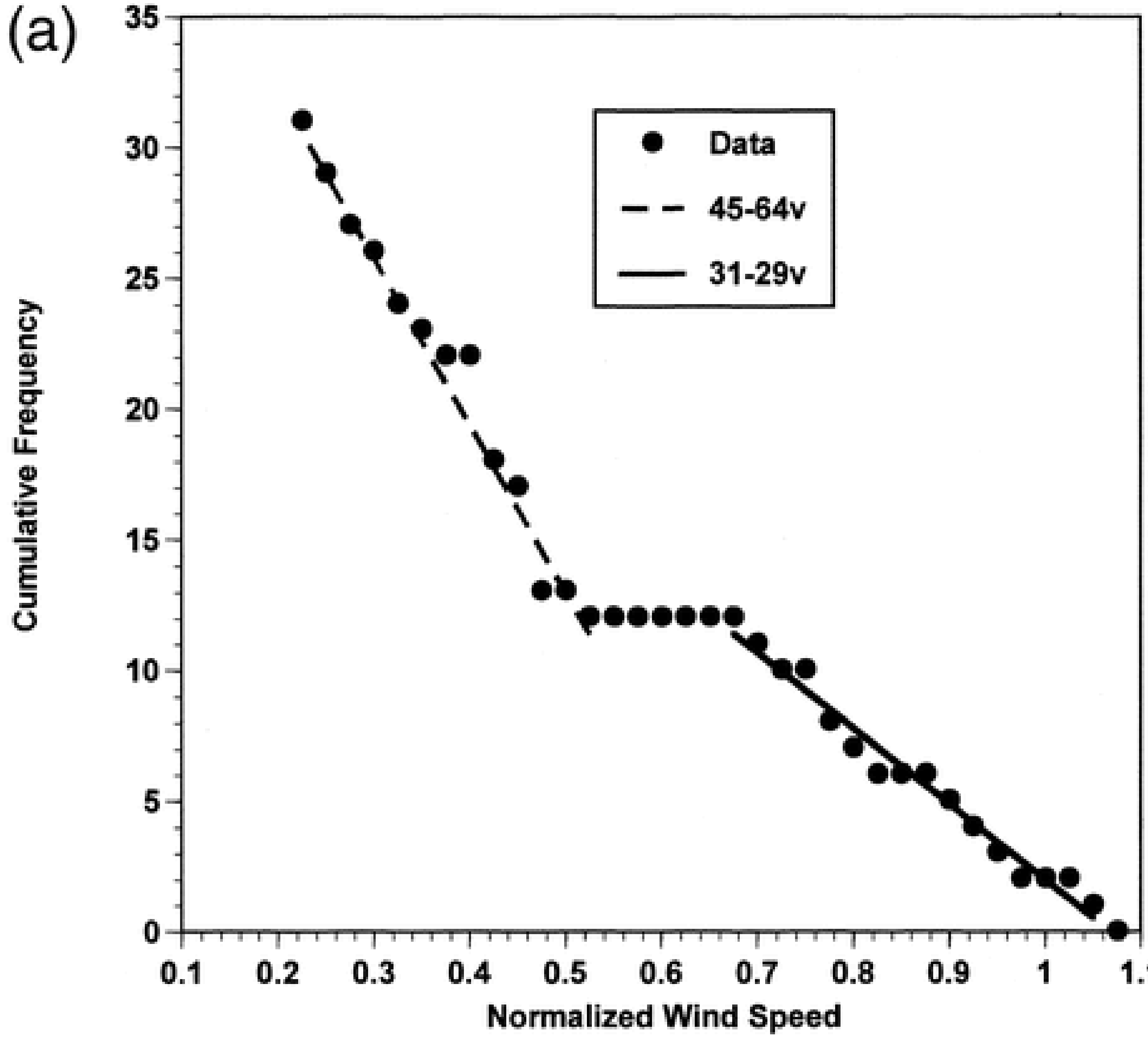
(a)



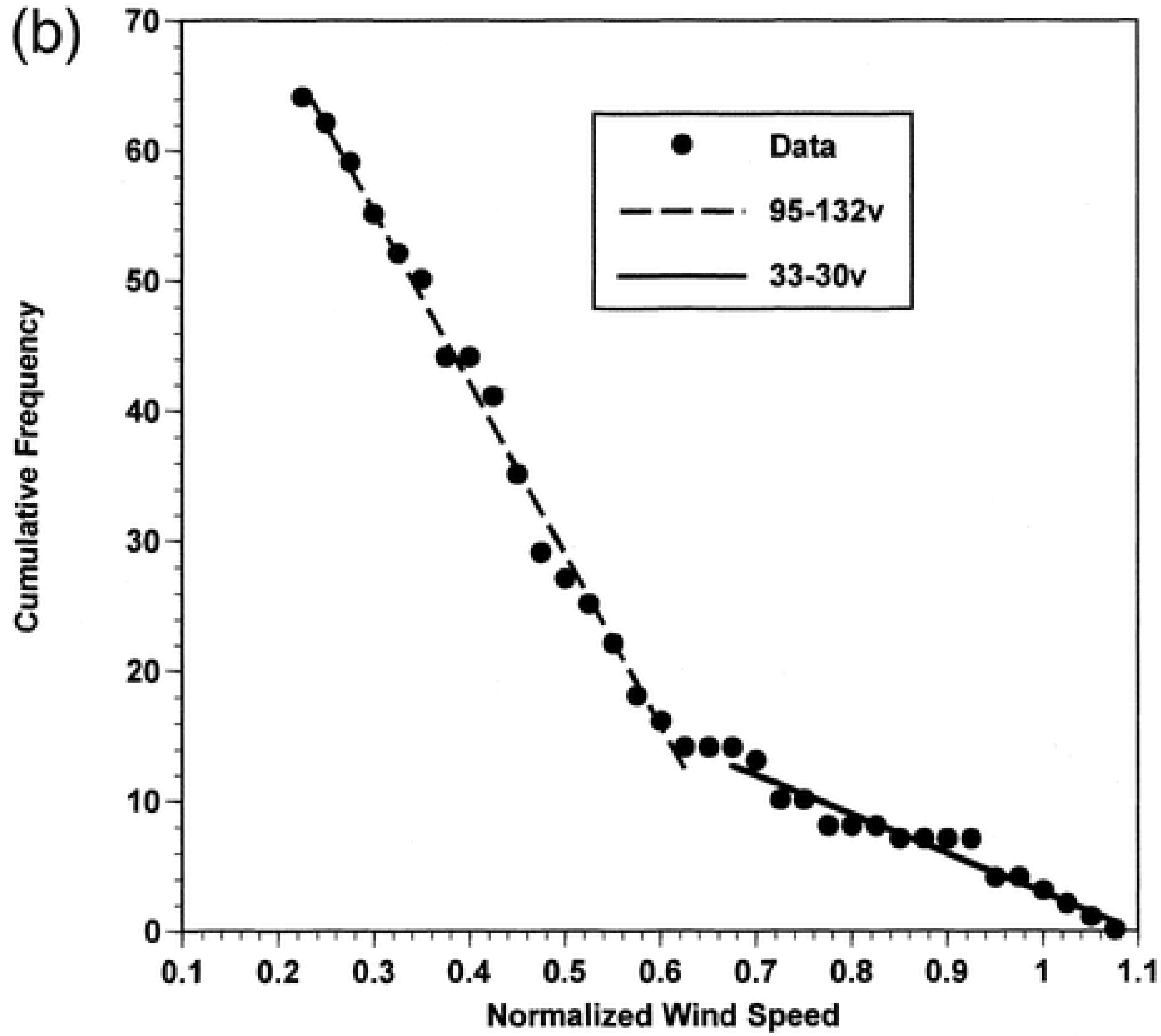
Evolution curve of WPAC storms whose lifetime maximum intensity is limited by declining potential intensity, but not by landfall



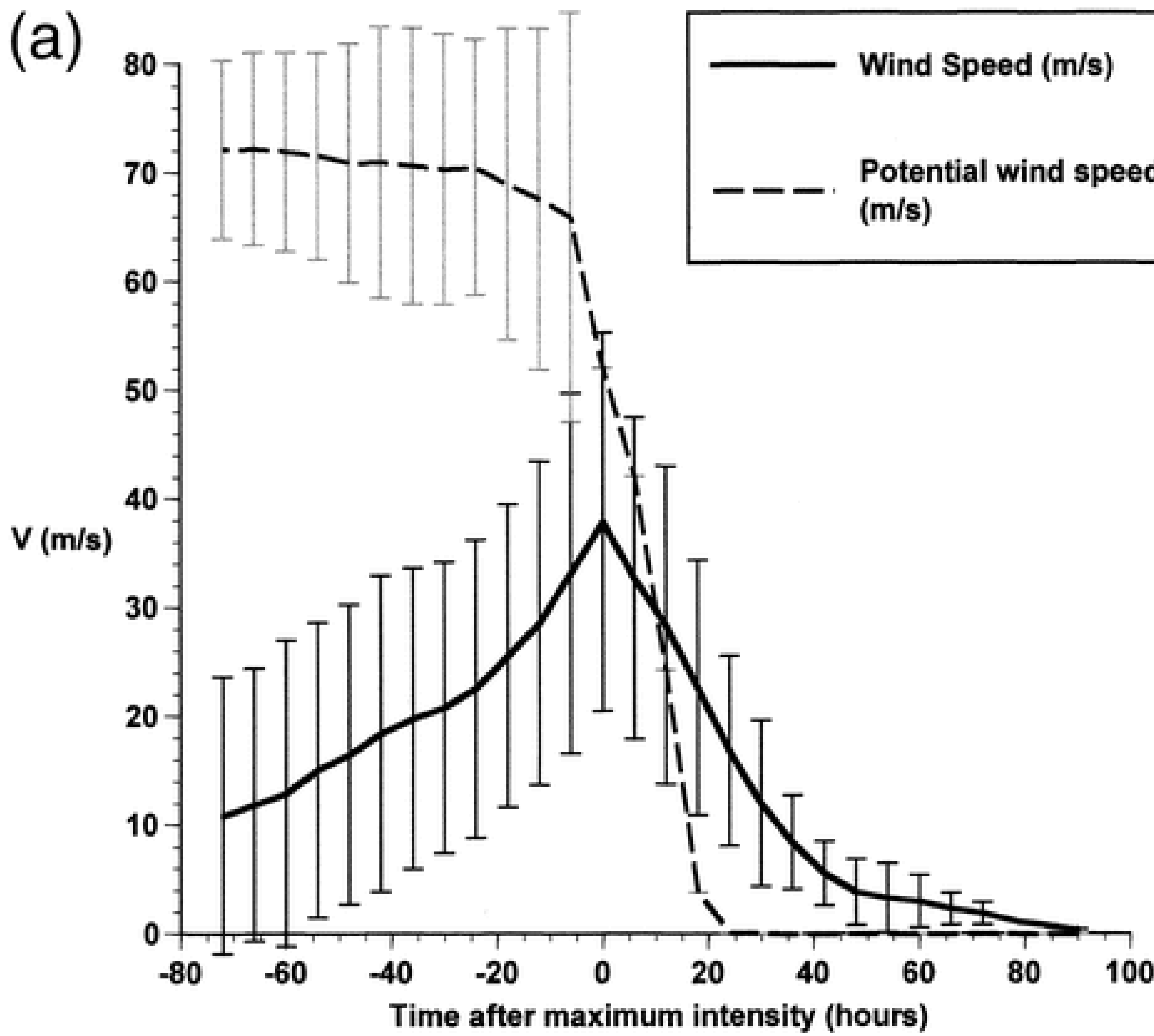
CDF of normalized lifetime maximum wind speeds of North Atlantic tropical cyclones of tropical storm strength (18 m s−1) or greater, for those storms whose lifetime maximum intensity was limited by landfall. (a)



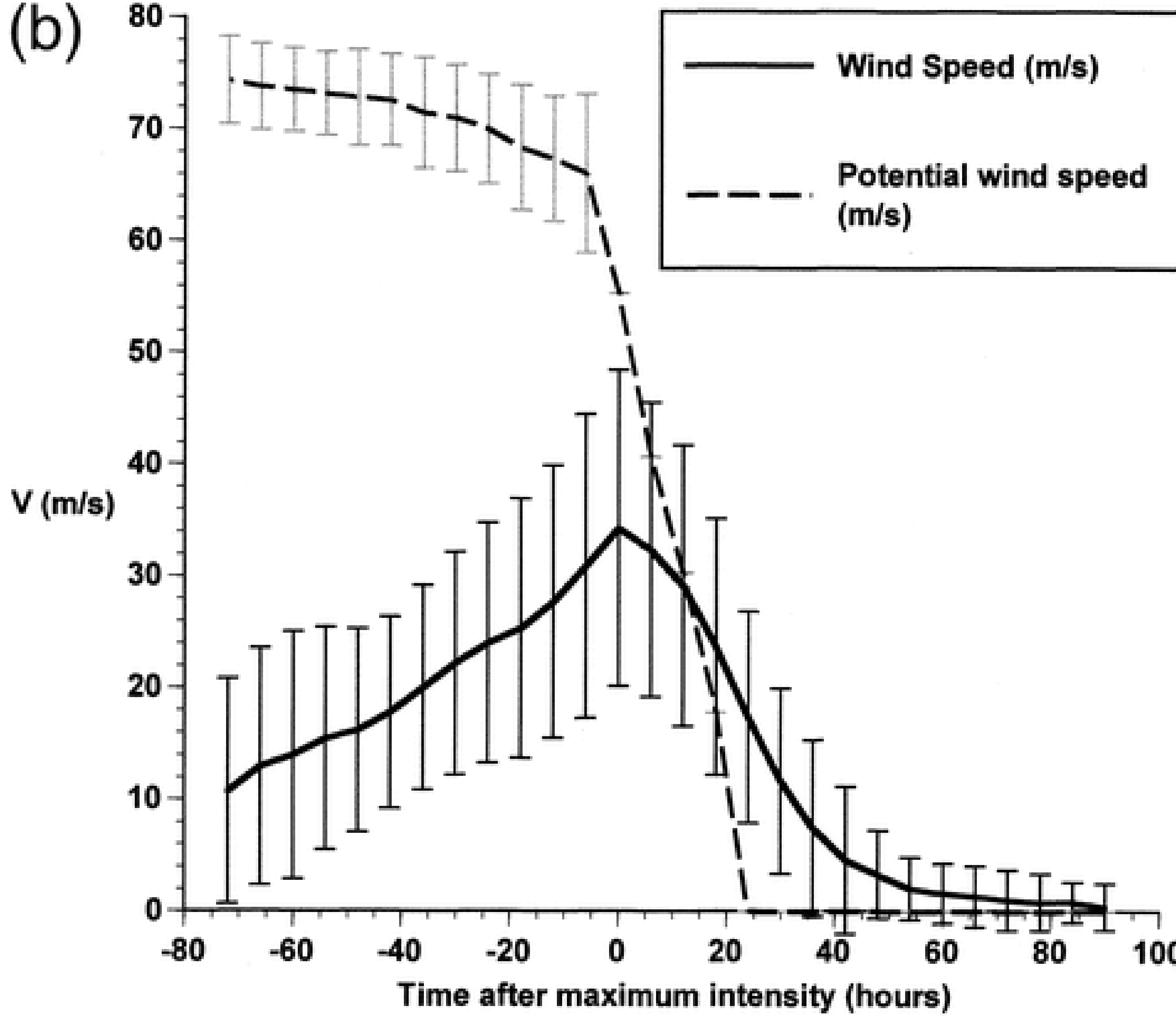
CDF of normalized lifetime maximum wind speeds of Northwest Pacific tropical cyclones of tropical storm strength (18 m s−1) or greater, for those storms whose lifetime maximum intensity was limited by landfall.



Evolution of Atlantic storms whose lifetime maximum intensity was limited by landfall



Evolution of Pacific storms whose lifetime maximum intensity was limited by landfall



Composite evolution of landfalling storms