# **Tropical Cyclone Motion**

Tropical cyclones move approximately with a suitably defined vertical vector average of the flow in which they are embedded









### Lagrangian chaos:



Vortices in PV gradients:



### Baroclinic vortices in shear: A simple model

- Two layers, with zero effective PV gradient, but upper layer moving with respect to lower layer
- Lower layer contains point potential vortex, whose circulation projects outward and upward
- Upper layer has point source of zero PV air co-located with lower point vortex; zero PV air separated from surroundings by a single, expanding contour
- Flow owing to upper level PV anomaly solved by contour dynamics

(From Wu and Emanuel, 1993)

#### Lower (left) and upper (right) flows for zero shear:



Evolution of upper layer votex patch when weak shear is present



### Evolution of upper layer votex patch when moderate shear is present



t = 3.0

# Evolution of upper layer votex patch when strong shear is present



y = 3.0



FIG. 11. Trajectories (units of 500 km) of the lower-layer vortex for  $\epsilon = 0.25$ ,  $\gamma = 0.79$ , and x = 0.25 (shown as "+"); x = 1.25 (shown as "•"); and x = 5 (shown as "O").

FIG. 12. The relation between the maximum induced vortex speed and the magnitude of the vertical shears ( $\chi$ ) for  $\epsilon = 0.25$  and  $\gamma = 0.79$ .

#### Operational prediction of tropical cyclone tracks:



year

# Example: 20 random tracks passing within 100 km of Boston



### 20 "worst" tracks:



Interaction of Tropical Cyclones with the Upper Ocean

- Resonance with near-inertial oscillations
- Mixed layer cooling by entrainment
- Coupled models



Change on SST needed to cancel increase in enthalpy in core:

$$L_{v}q^{*}(T_{a})H + c_{p}T_{a} = L_{v}q^{*}(T_{a} - \Delta T) + c_{p}(T_{a} - \Delta T)$$
$$L_{v}q^{*}(T_{a} - \Delta T) \cong L_{v}q^{*}(T_{a}) - L_{v}\frac{\partial q^{*}}{\partial T}\Delta T$$
$$= L_{v}q^{*}(T_{a}) - L_{v}\frac{L_{v}q^{*}}{R_{v}T_{a}^{2}}\Delta T$$

$$\rightarrow \Delta T \cong \frac{L_v q^* (1-H)}{c_p + \frac{L_v^2 q^*}{R_v T_a^2}} \cong 2.5^o C$$

Physics of near-inertial oscillations:

PEs linearized about a rotating stratified fluid at rest:

 $\frac{\partial u}{\partial t} = -\alpha_0 \frac{\partial p}{\partial x} + fv$  $\frac{\partial v}{\partial t} = -\alpha_0 \frac{\partial p}{\partial y} - fu$  $\frac{\partial w}{\partial t} = -\alpha_0 \frac{\partial p}{\partial z} + B$  $\frac{\partial B}{\partial t} = -N^2 w$  $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ 

$$\rightarrow \frac{\partial^2}{\partial t^2} \nabla_3^2 w + N^2 \nabla_2^2 w + f^2 \frac{\partial^2 w}{\partial t^2} = 0$$

$$w = w_0 e^{i(kx + ly + rz - \omega t)}$$





Mixing and Entrainment:



### **Entrainment Formulation:**

Criticality of a Bulk Richardson Number:

$$Ri = \frac{gh\Delta\rho}{\rho u^2}$$

Assume that density jump is what would result from eroding a constant background stratification down to depth h:

$$Ri \equiv \frac{1}{2} \frac{h^2 N^2}{u^2}$$
  
Equivalently,  $\boxed{Nh = R'u}$  (1)  
 $R' \equiv \sqrt{2Ri}$ 

Criticality assumption: *R*' = *constant*.

Mixed layer momentum conservation (neglecting Coriolis turning) :

$$\frac{\partial \left(\rho_{w} u h\right)}{\partial t} = \rho_{a} u_{*}^{2}.$$
<sup>(2)</sup>

$$u_*^2 \equiv C_D |\mathbf{V}|^2$$

Combine (2) with (1):

$$\frac{\partial h^2}{\partial t} = R' \frac{\rho_a}{\rho_w} \frac{u_*^2}{N}$$

Note: units of diffusivity

#### Comparison with same atmospheric model coupled to 3-D ocean model; idealized runs: Full model (black), string model (red)



### Mixed layer depth and currents

Full physics coupled run ML depth (m) and currents at t=10 days







# SST Change

Full physics coupled run  $\triangle$  SST (<sup>o</sup>C) at t=10 days



#### Independent columns coupled run $\Delta$ SST (<sup>o</sup>C) at t=10 days



(a) Mixed-layer depth on the axis of the storm's motion (m)



Define feedback factor:

$$F_{SST} = \frac{\Delta p}{\Delta p \mid_{SST}} - 1,$$

where  $\Delta p \mid_{SST}$  is the central pressure drop at fixed SST. Do many, many numerical expreiments, varying SST, Coriolis parameter, traslantion speed, etc. Curve fit dependence of  $F_{SST}$  on these parameters. Result:



HURRICANE TRANSLATION SPEED (m/s)

 $F_{SST} = -0.87e^{-z}$ 

$$z = 0.55 \left(\frac{h_0}{30 \ m}\right)^{1.04} \left(\frac{u_T}{6 \ m \ s^{-1}}\right)^{0.97} \left(\frac{\Delta p \mid_{SST}}{50 \ hPa}\right)^{-0.78} \times \eta^{-0.85} \left(\frac{f}{5 \times 10^{-5} \ s^{-1}}\right)^{0.59} \left(\frac{\Gamma}{8 \times 10^{-2} \ K \ m^{-1}}\right)^{-0.40} \left(\frac{1 - \mathcal{H}}{0.2}\right)^{0.46}$$

 $\eta$ = storm size scaling factor



### Effects of Environmental Wind Shear

- Dynamical effects
- Thermodynamic effects
- Net effect on intensity





PV dynamics



Streamlines (dashed) and  $\theta$  surfaces (solid)



#### Wind Speed (m/s) at 84 h



Wind Speed (m/s) at 60 h

