Above a thin boundary layer, most atmospheric convection involved phase change of water:

Moist Convection

Moist Convection

- Significant heating owing to phase changes of water
- Redistribution of water vapor – most important greenhouse gas
- Significant contributor to stratiform cloudiness – albedo and longwave trapping

Water Variables

Mass concentration of water vapor (*specific humidity*):

$$q \equiv M_{H_2O} / M_{air}$$

Vapor pressure (partial pressure of water vapor): *e*

Saturation vapor pressure: *e**

C-C: $$e^* = 6.112\, hPa\, e^{\frac{17.67(T-273)}{T+30}}$$

Relative Humidity: $$\mathcal{H} \equiv e / e^*$$

The Saturation Specific Humidity

Ideal Gas Law:

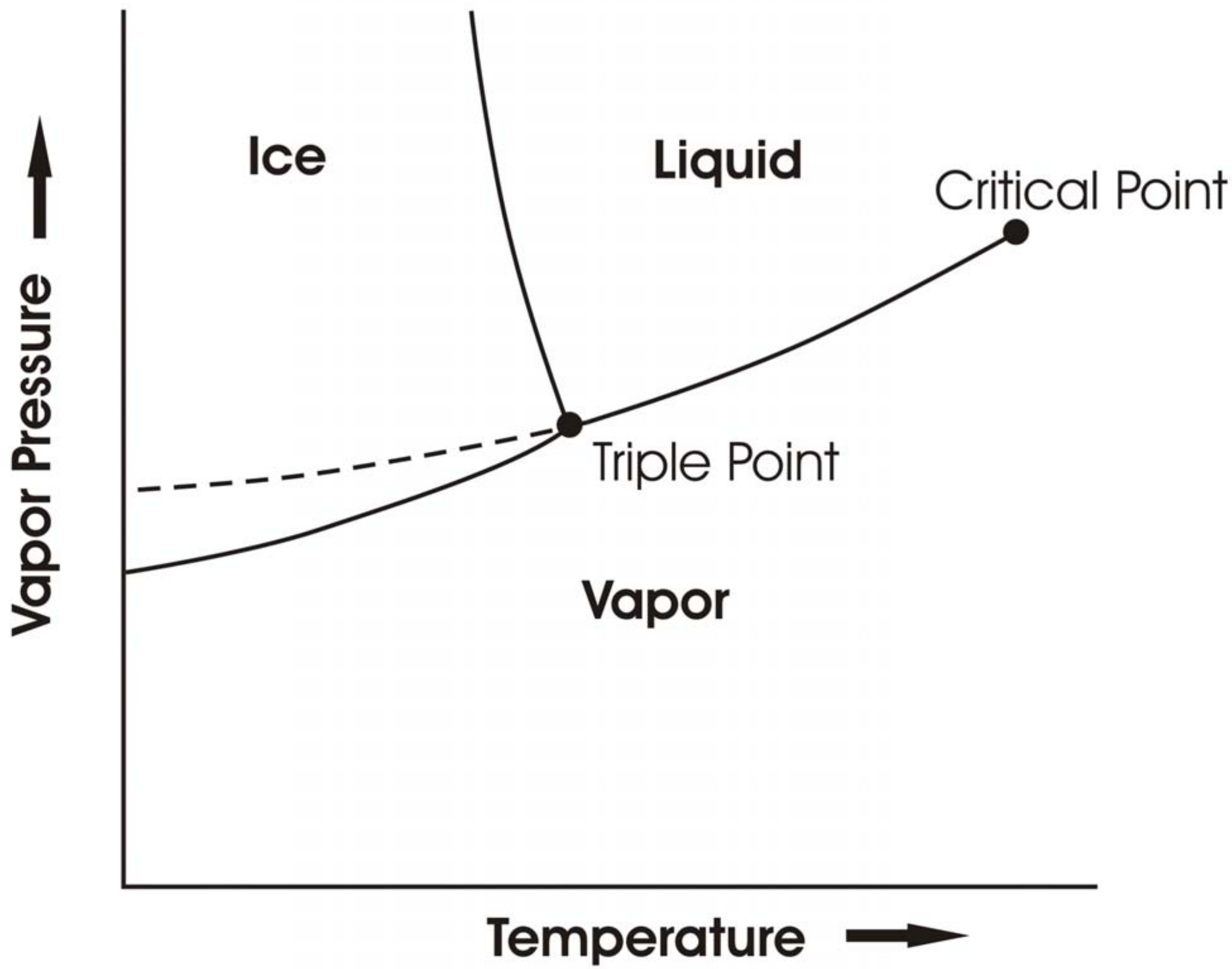
$$p = \rho \frac{R^* T}{\bar{m}}$$

$$e = \rho_v \frac{R^* T}{m_v}$$

$$q = \rho_v / \rho = \frac{m_v}{\bar{m}} \frac{e}{p}$$

$$q^* = \frac{m_v}{\bar{m}} \frac{e^*}{p}$$

Phase Equilibria



Bringing Air to Saturation

$$e = qp\left(\bar{m}\big/m_v\right)$$

$$e^* = e^*(T)$$

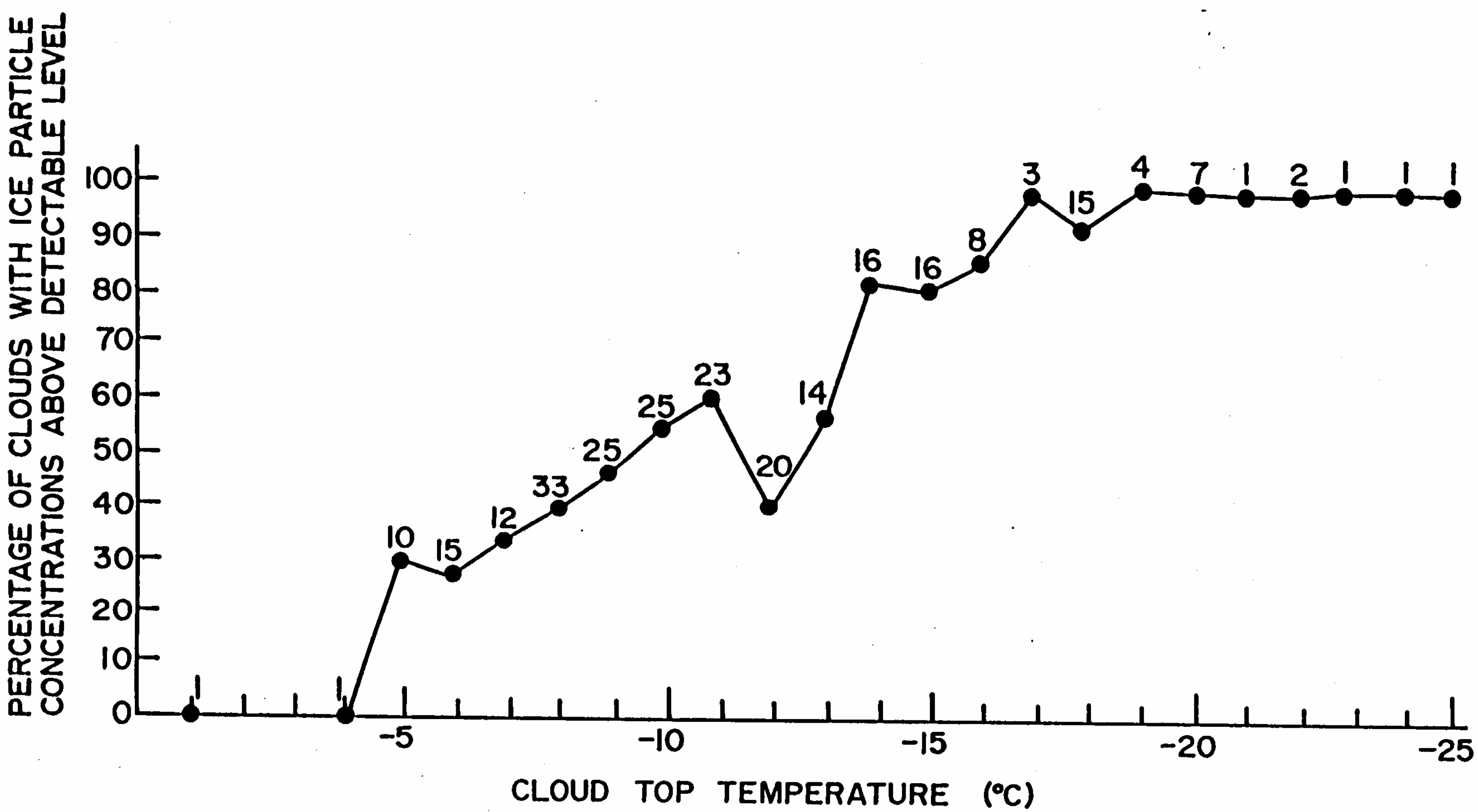
1. Increase *q (or p)*

2. Decrease *e* (T)*

When Saturation Occurs…

- Heterogeneous Nucleation
- Supersaturations very small in atmosphere
- Drop size distribution sensitive to size distribution of cloud condensation nuclei

Ice Nucleation Problematic



Precipitation Formation:

- Stochastic coalescence (sensitive to drop size distributions)
- Bergeron-Findeisen Process
- Strongly nonlinear function of cloud water concentration
- Time scale of precipitation formation ~10-30 minutes

Stability

No simple criterion based on entropy:

$$s_d = c_p \ln\left(\frac{T}{T_0}\right) - R_d \ln\left(\frac{p}{p_0}\right)$$

$$\alpha = \alpha\left(s_d, p\right)$$

$$s = c_p \ln\left(\frac{T}{T_0}\right) - R_d \ln\left(\frac{p}{p_0}\right) + L_v \frac{q}{T} - qR_v \ln\left(\mathcal{H}\right)$$

$$\alpha = \alpha\left(s, p, q_t\right)$$

Virtual Temperature and Density Temperature

Assume all condensed water falls at terminal velocity

$$\alpha = \frac{V_a + V_c}{M_d + M_v + M_c}$$

$$pV = nR^*T$$

$$V_a = \frac{R^*T}{p}\left(\frac{M_d}{m_d} + \frac{M_v}{m_v}\right),$$

$$\overline{m_d} \equiv \frac{1}{\frac{1}{M_d}\sum_i \frac{M_i}{m_i}}$$

$$\rightarrow \quad V_a = \frac{R_d T}{p}\left(M_d + M_v/\varepsilon\right),$$

where $$\varepsilon \equiv m_v/m_d \cong 0.622$$

$$R_d \equiv R^*/m_d$$

$$\alpha = \frac{V_a + V_c}{M_d + M_v + M_c} = \frac{R_d T}{p}\left(1 - q_t + q/\varepsilon\right)\left(1 + \frac{q_c}{1 - q_c}\frac{\rho_a}{\rho_c}\right)$$

$$\cong \frac{R_d T}{p}\left(1 - q_t + q/\varepsilon\right)$$

$$q_t \equiv \frac{M_v + M_c}{M}, \qquad q \equiv \frac{M_v}{M}$$

Density temperature:

$$T_\rho \equiv T\left(1 - q_t + q/\varepsilon\right)$$

$$\alpha = \frac{R_d T_\rho}{p}$$

Trick:

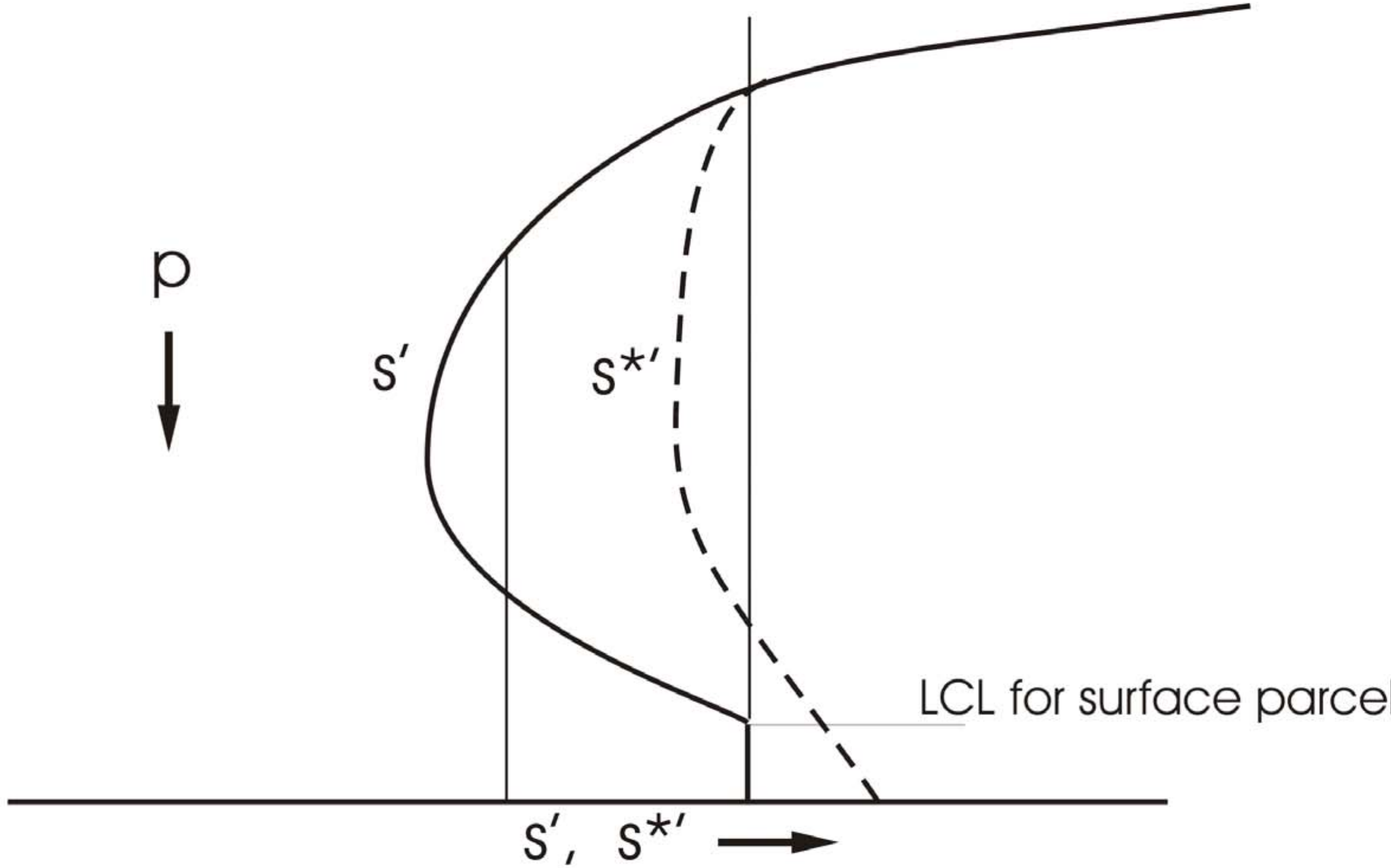
Define a *saturation entropy, s** :

$$s^* \equiv s\left(T, p, q^*\right)$$

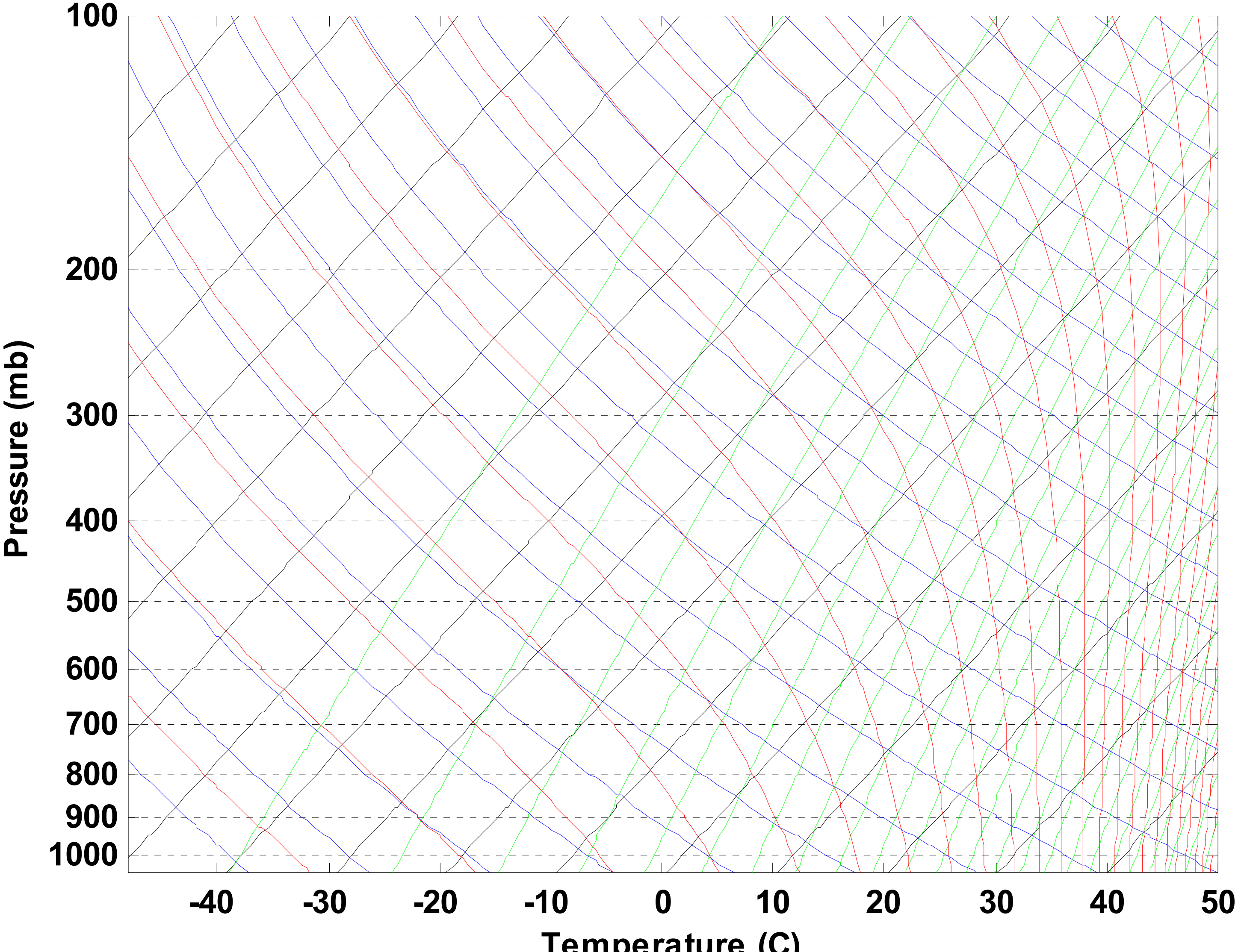
$$\alpha = \alpha\left(s^*, p, q_t\right)$$

We can add an arbitrary function of q_t to s^* such that

$$\alpha \cong \alpha\left(s^{*\prime}, p\right)$$


p
s′
s*′
LCL for surface parcel
s′, s*′

Stability Assessment using Tephigrams:

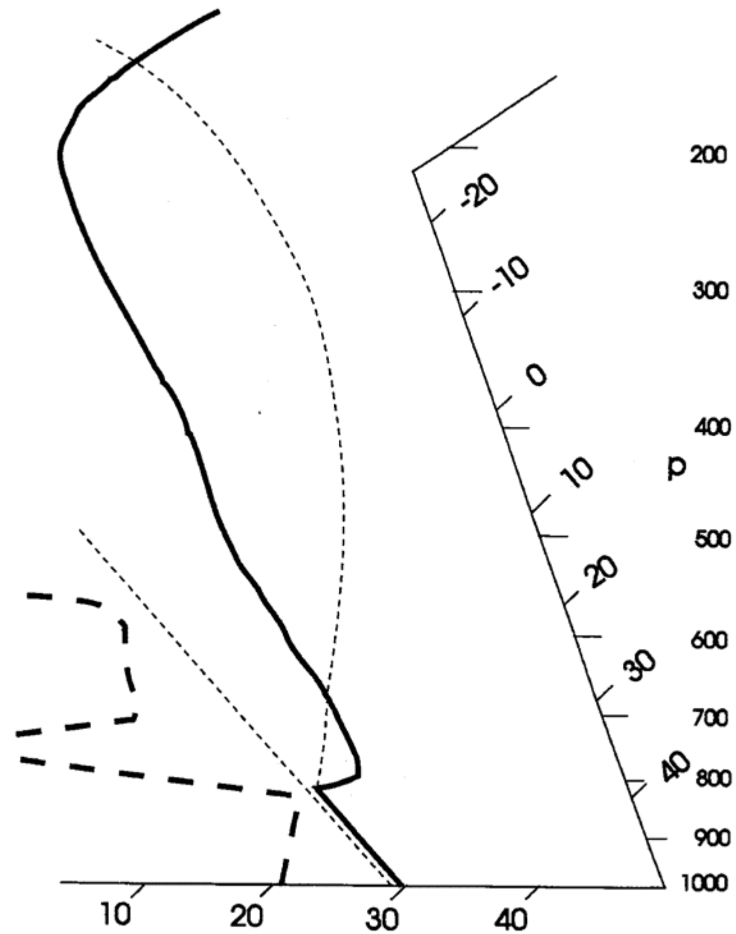


Stability Assessment using Tephigrams:

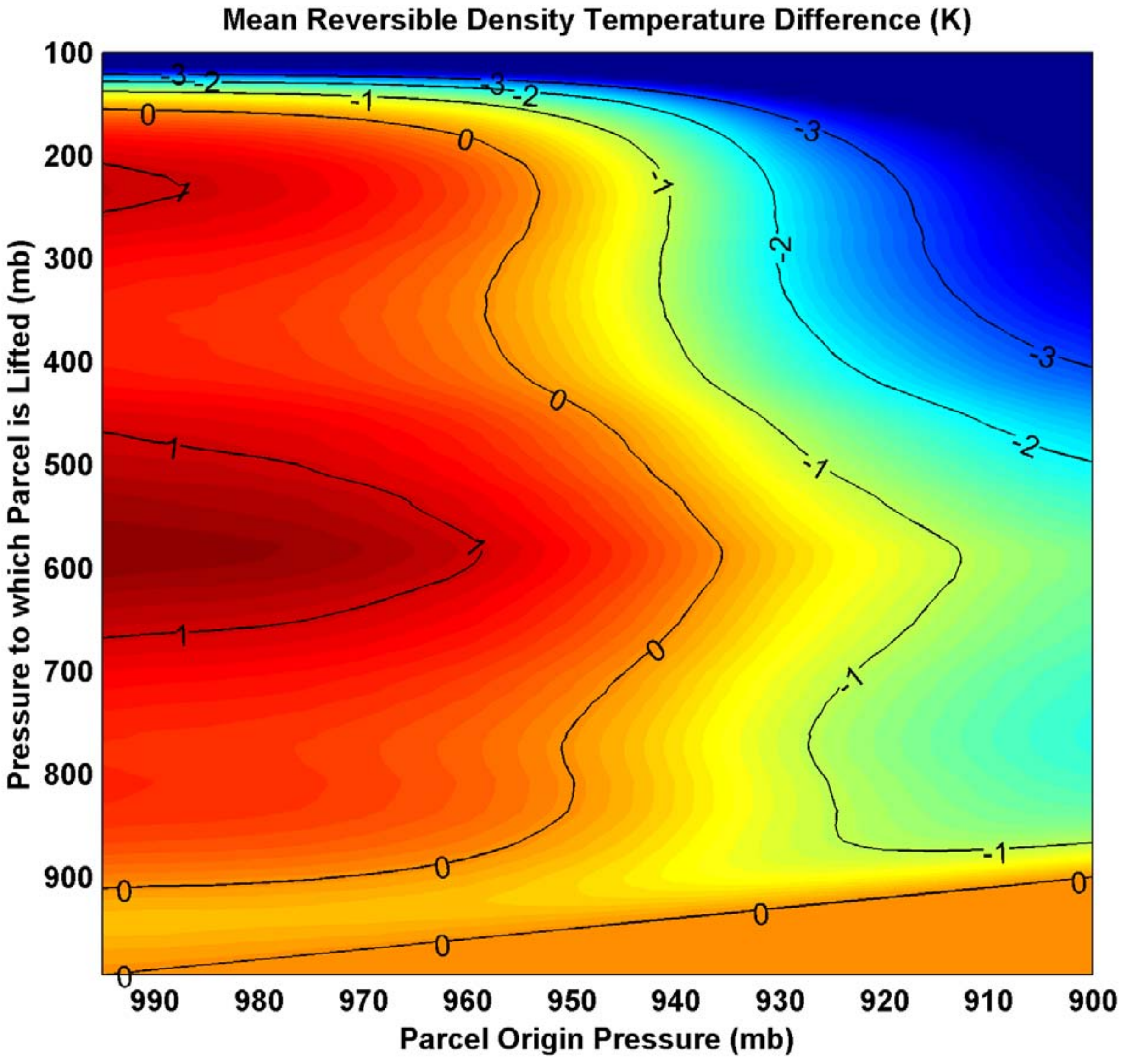
Convective Available Potential Energy (CAPE):

$$CAPE_i \equiv \int_{p_n}^{p_i} \left(\alpha_p - \alpha_e\right) dp$$

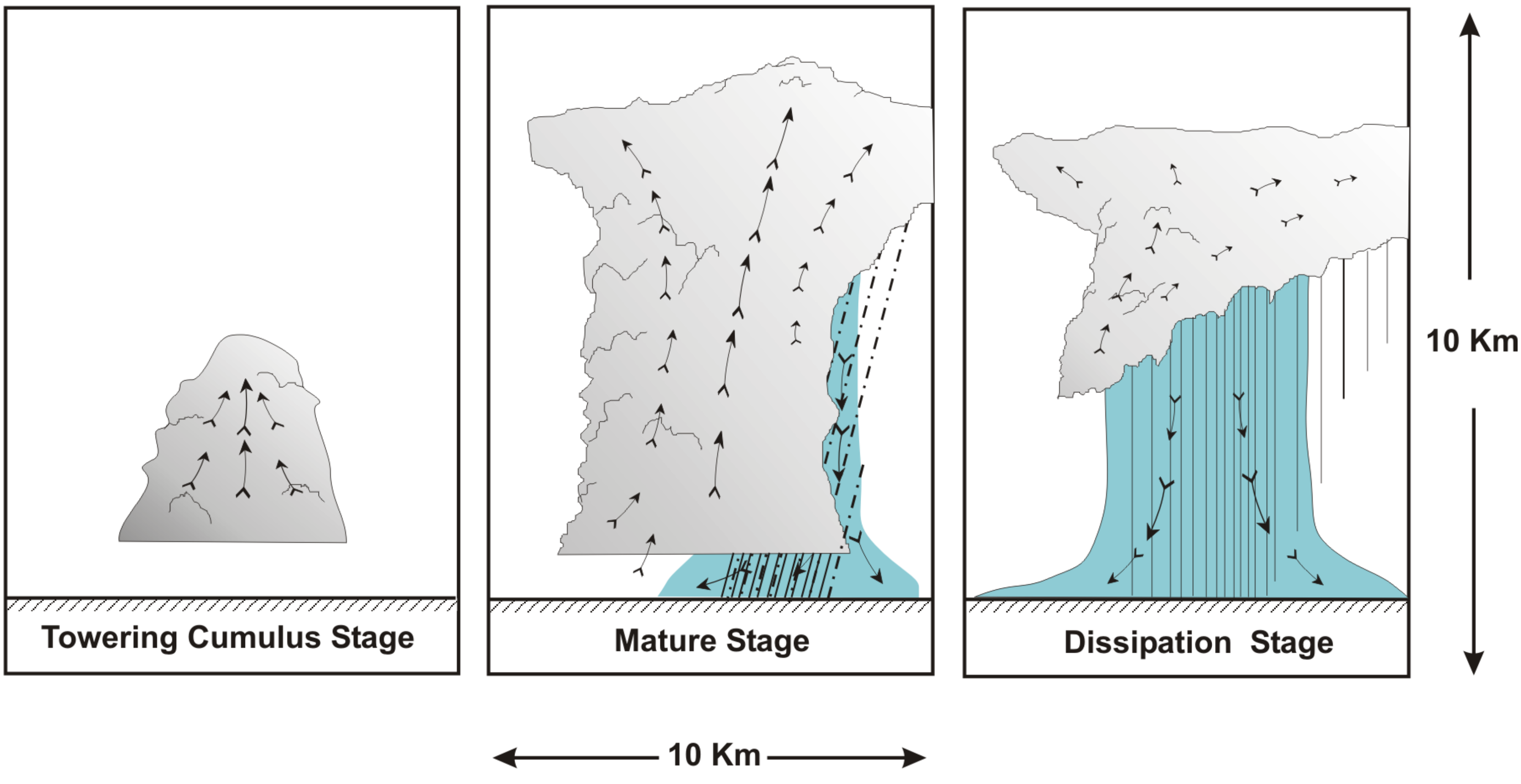
$$= \int_{p}^{p_i} R_d \left(T_{\rho_p} - T_{\rho_e}\right) d\ln\left(p\right)$$



Other Stability Diagrams:



“Air-Mass” Showers:



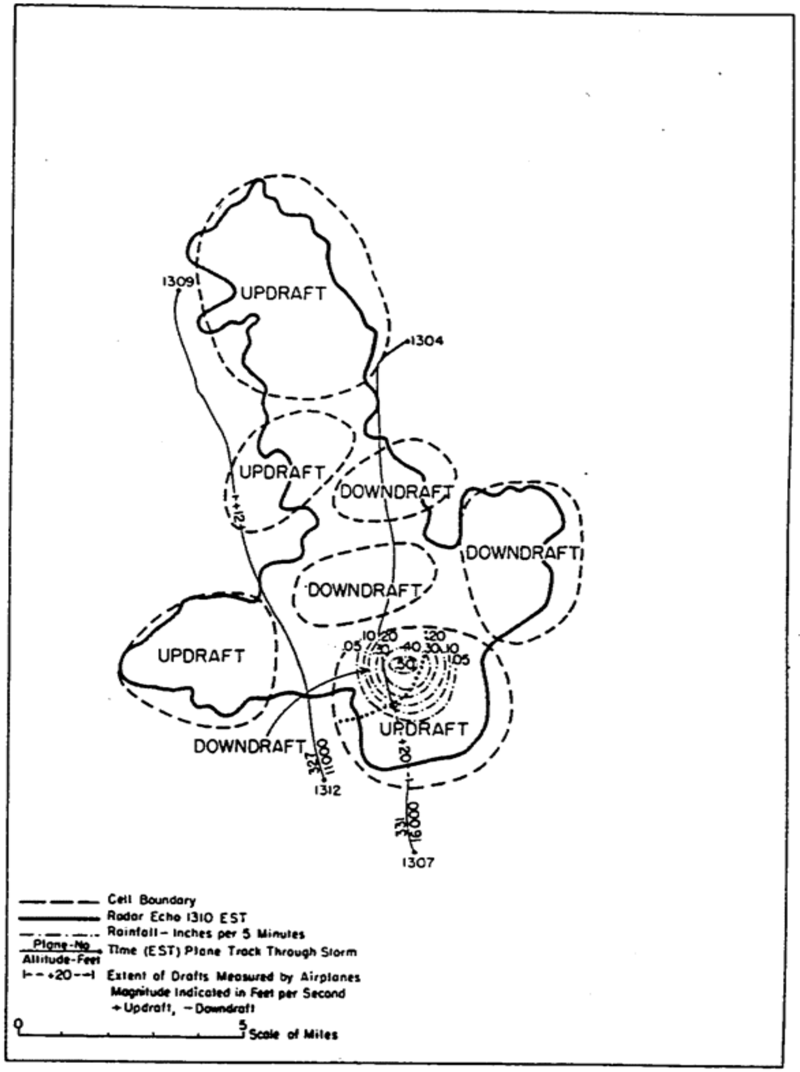


FIG. 15. Radar echo, plane paths, measured draft data, and cell outlines, 1310 EST 9 July 1946.

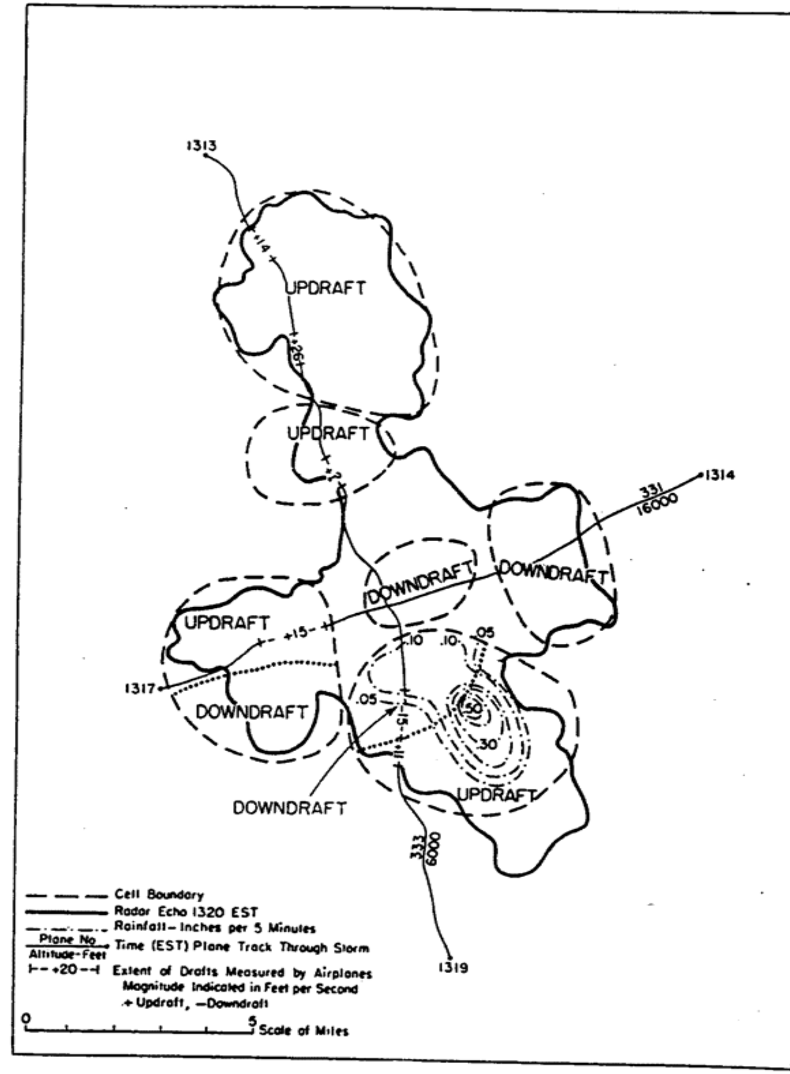
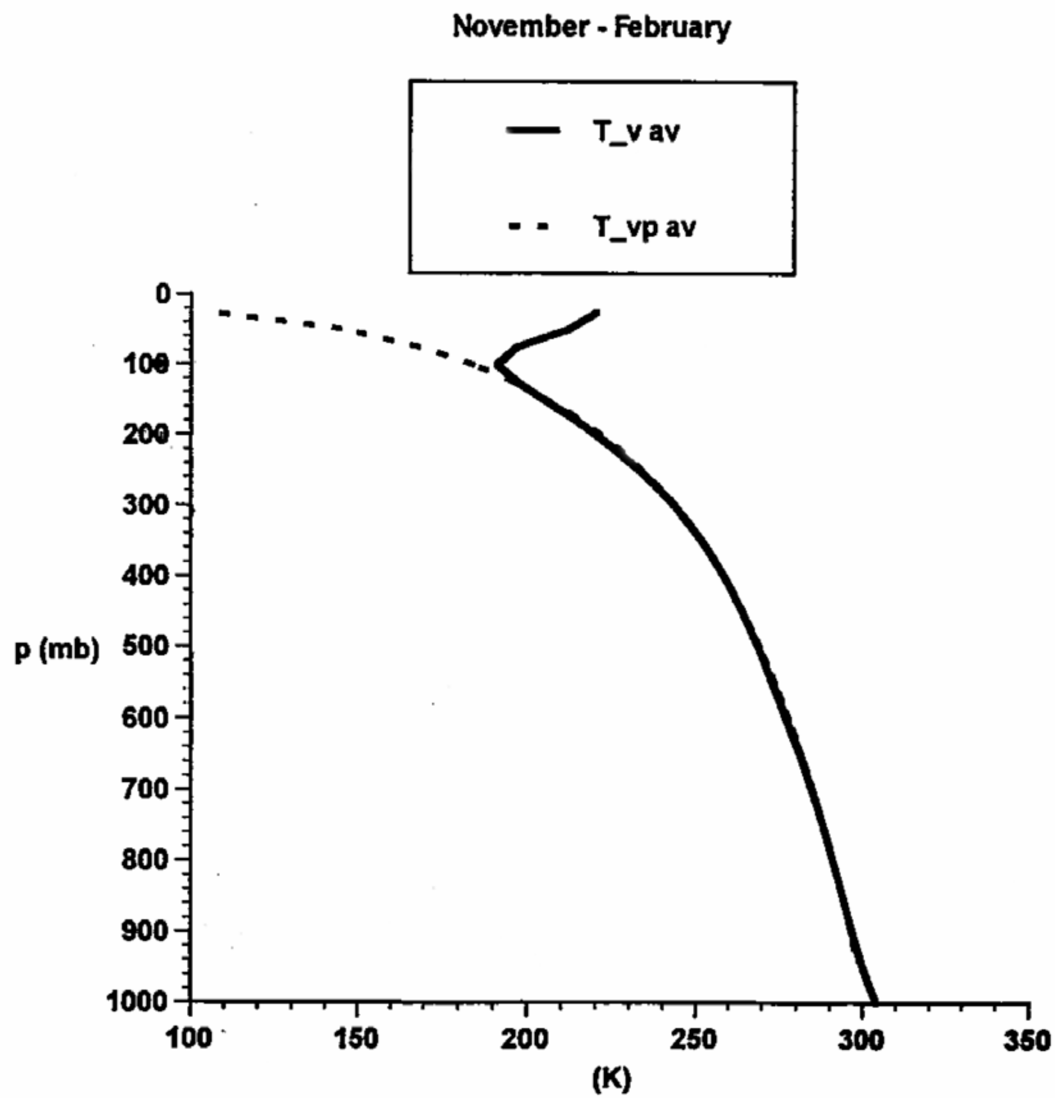
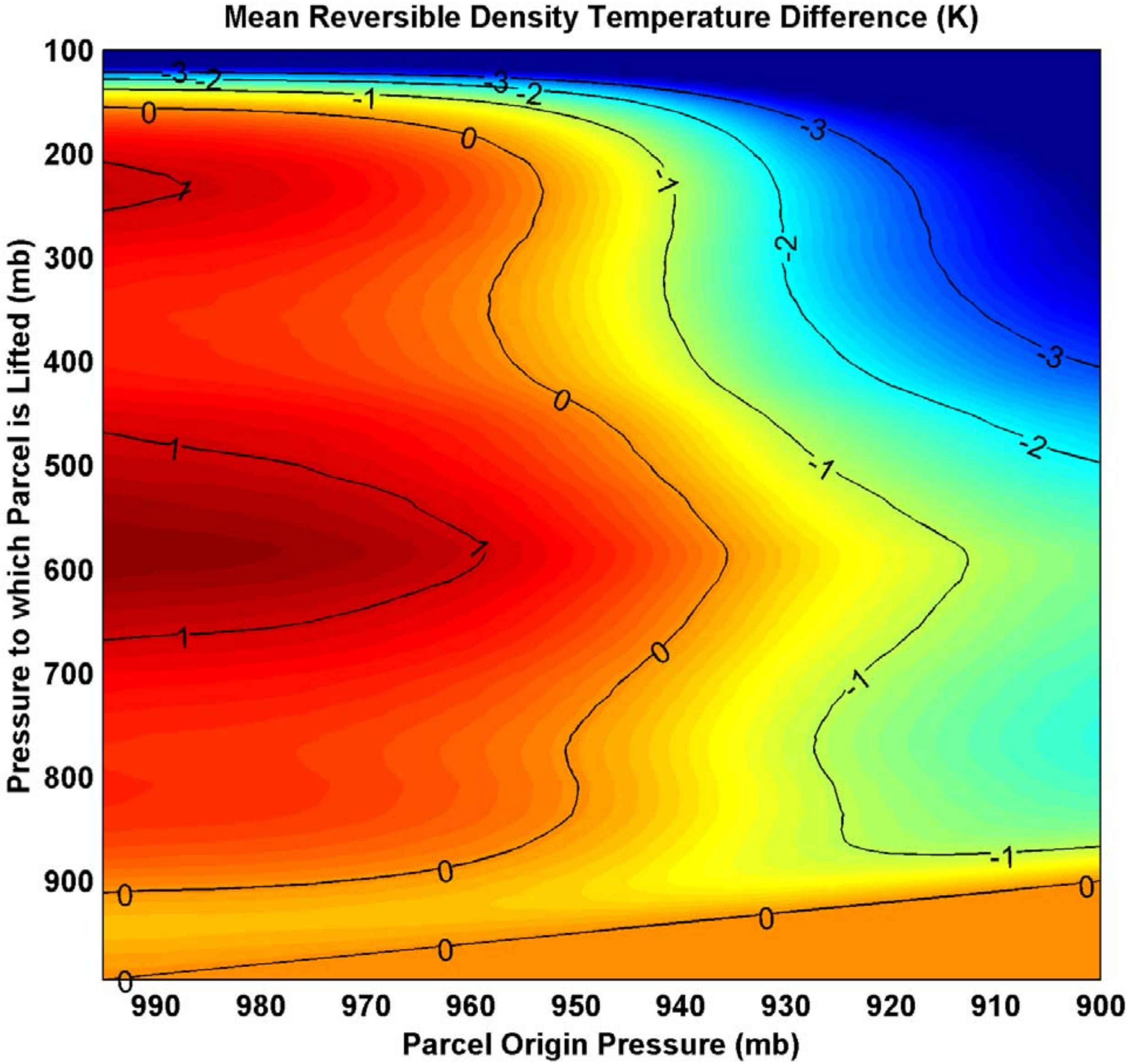


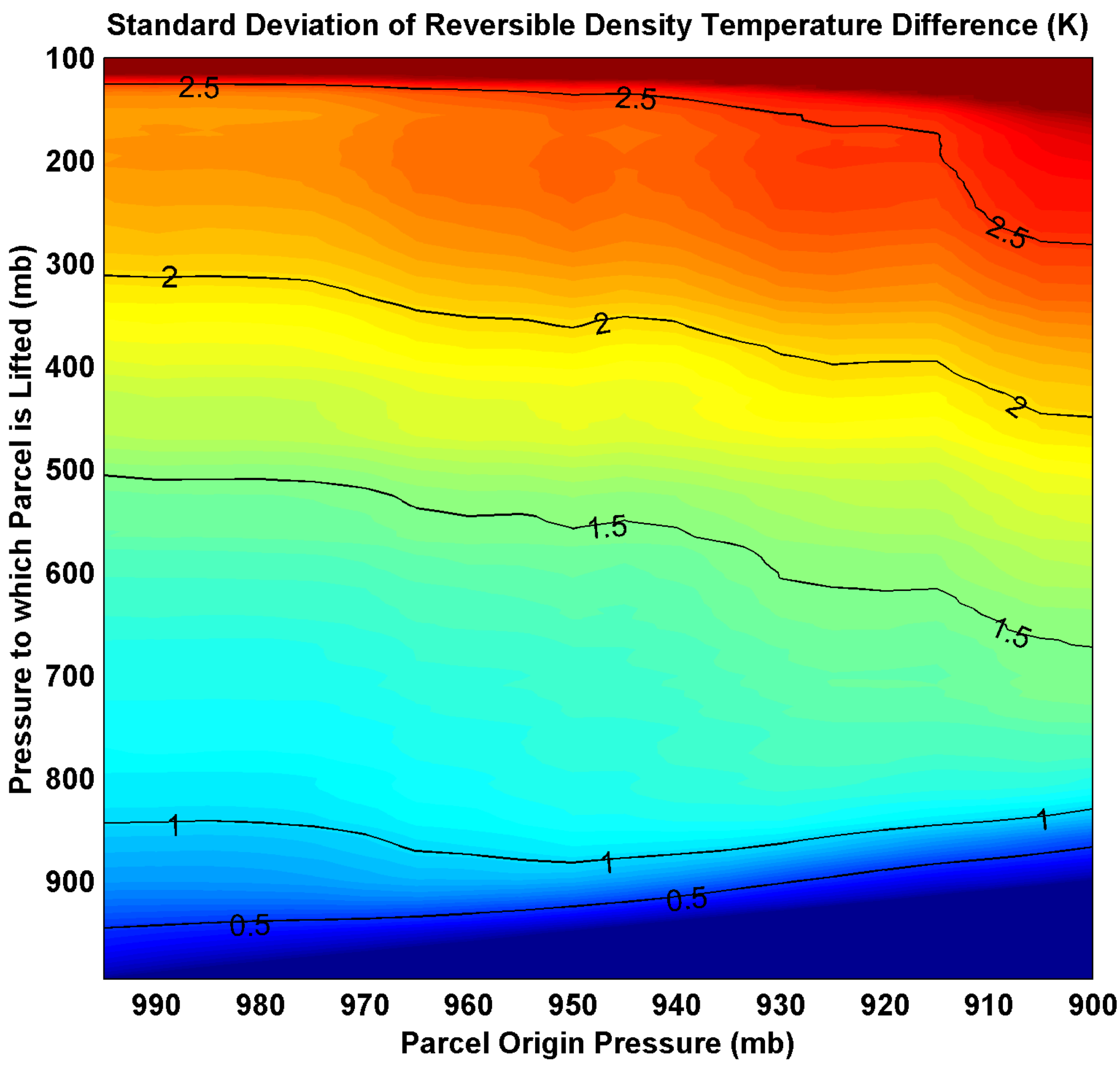
FIG. 16. Radar echo, plane paths, measured draft data, and cell outlines, 1320 EST 9 July 1946.

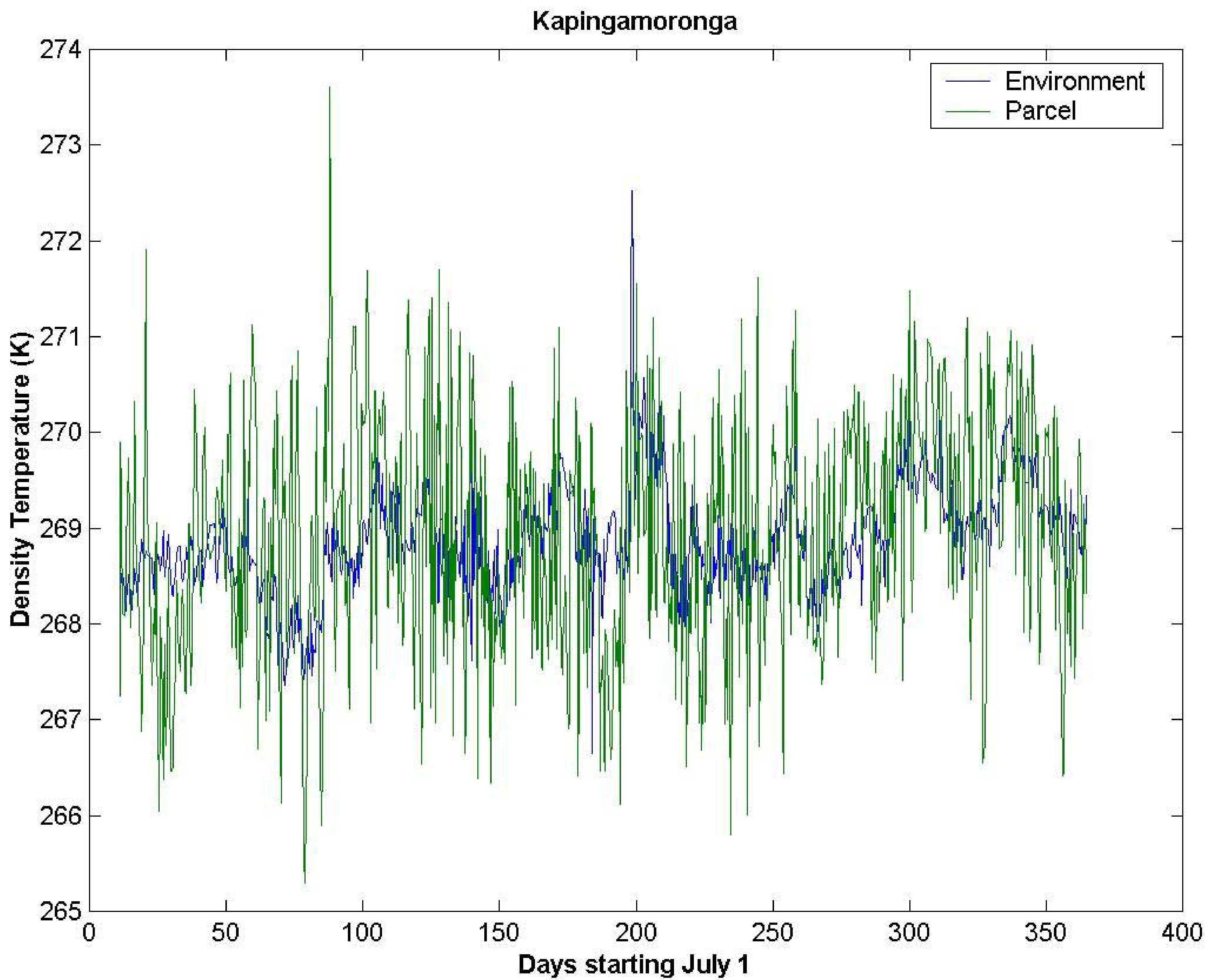
Tropical Soundings



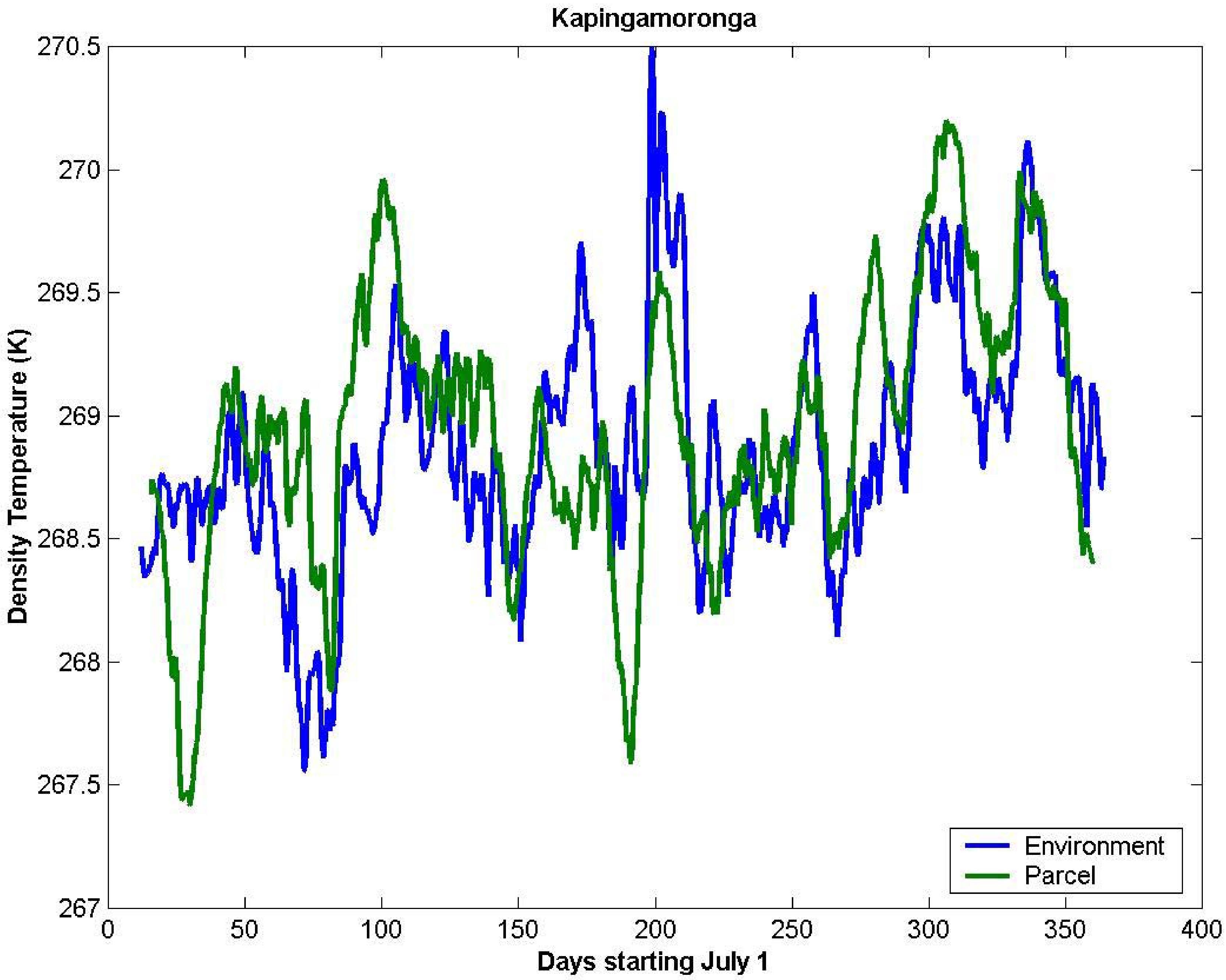
Annual Mean Kapingamoronga:

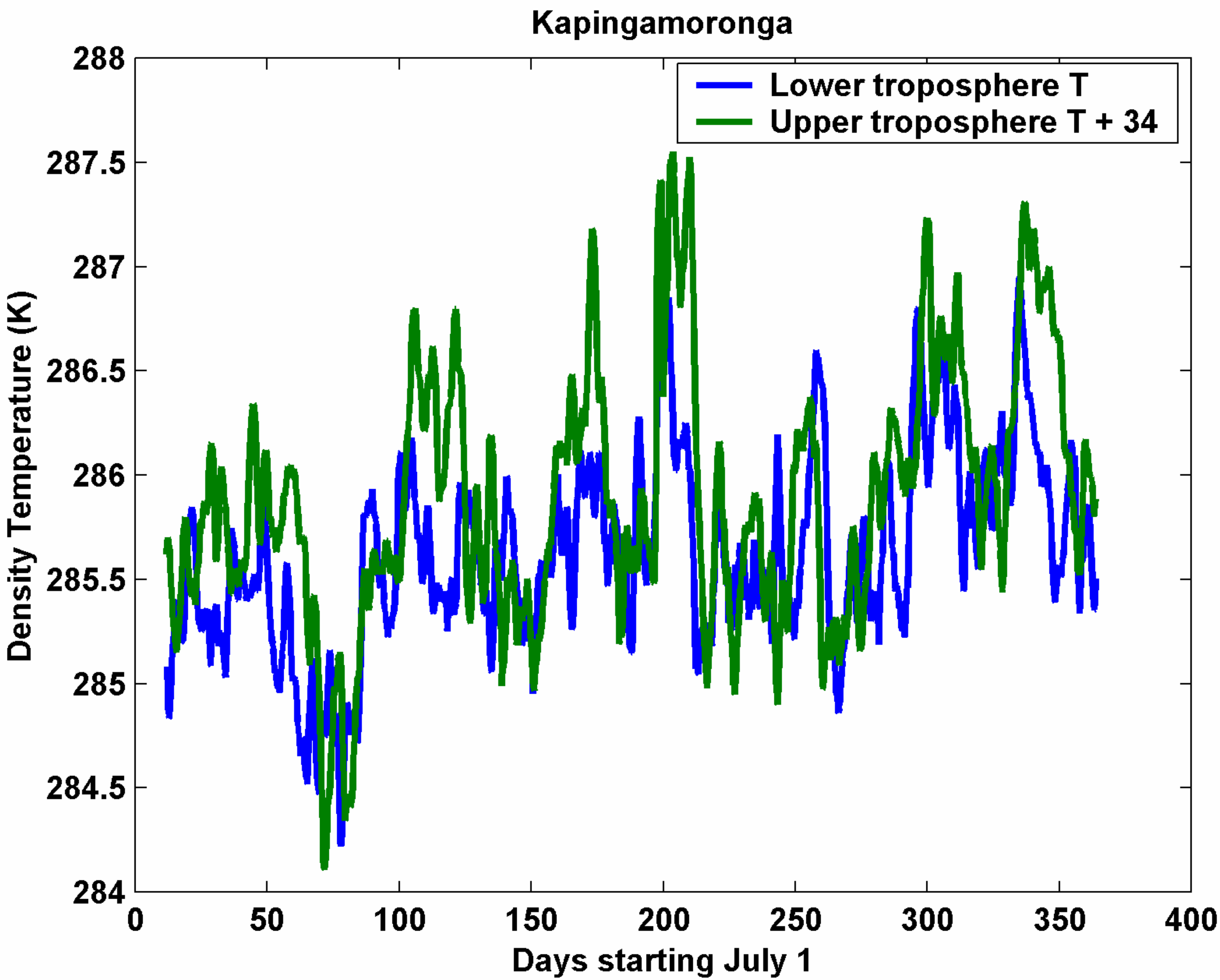



Standard Deviation of Reversible Density Temperature Difference (K)
Pressure to which Parcel is Lifted (mb)
Parcel Origin Pressure (mb)
100
200
300
400
500
600
700
800
900
990
980
970
960
950
940
930
920
910
900
2.5
2.5
2.5
2
2
2
1.5
1.5
1
1
1
0.5
0.5

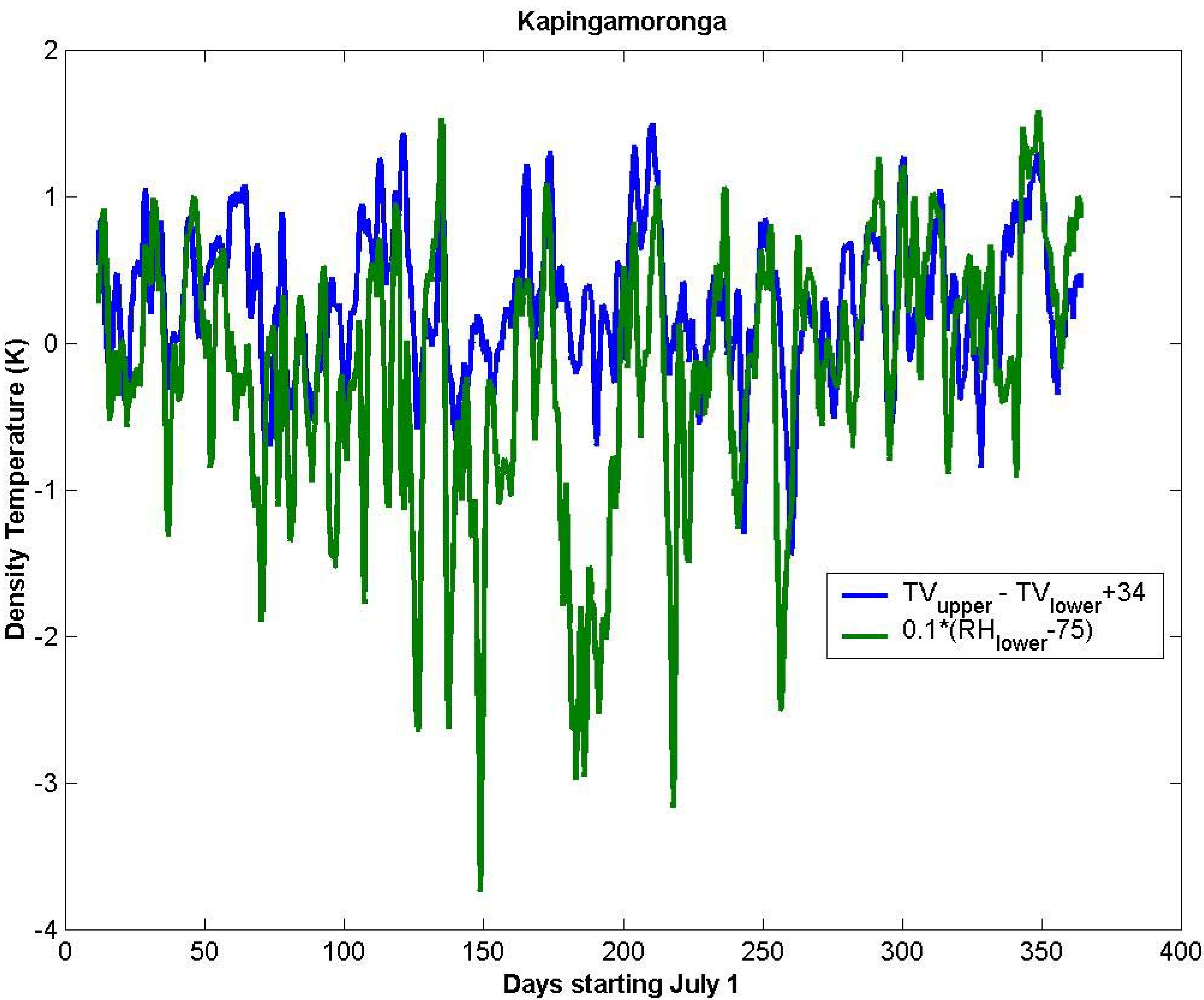


Kapingamoronga
Environment
Parcel
Density Temperature (K)
274
273
272
271
270
269
268
267
266
265
0
50
100
150
200
250
300
350
400
Days starting July 1

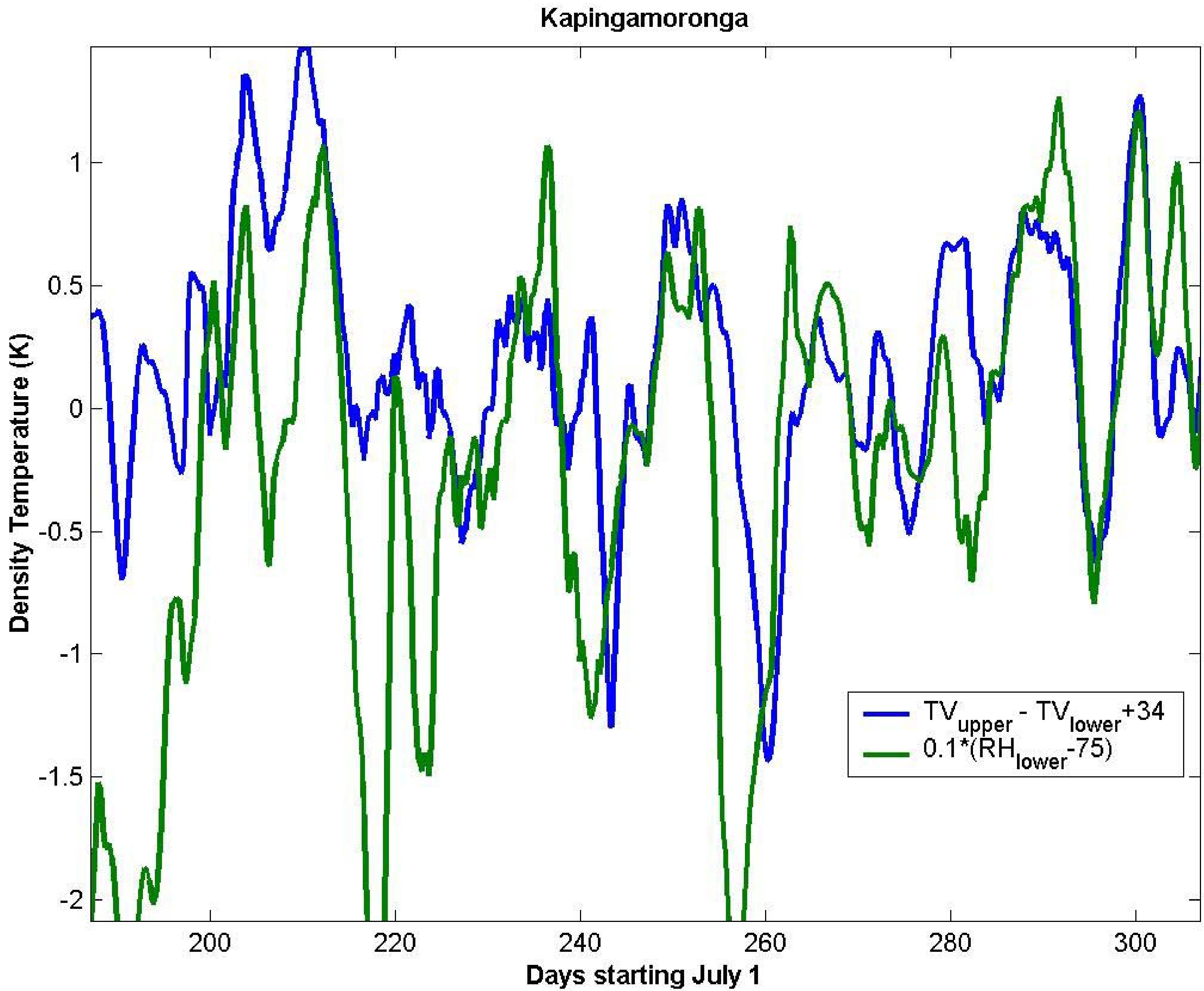

Kapingamoronga
270.5
270
269.5
269
268.5
268
267.5
267
Density Temperature (K)
0
50
100
150
200
250
300
350
400
Days starting July 1
Environment
Parcel

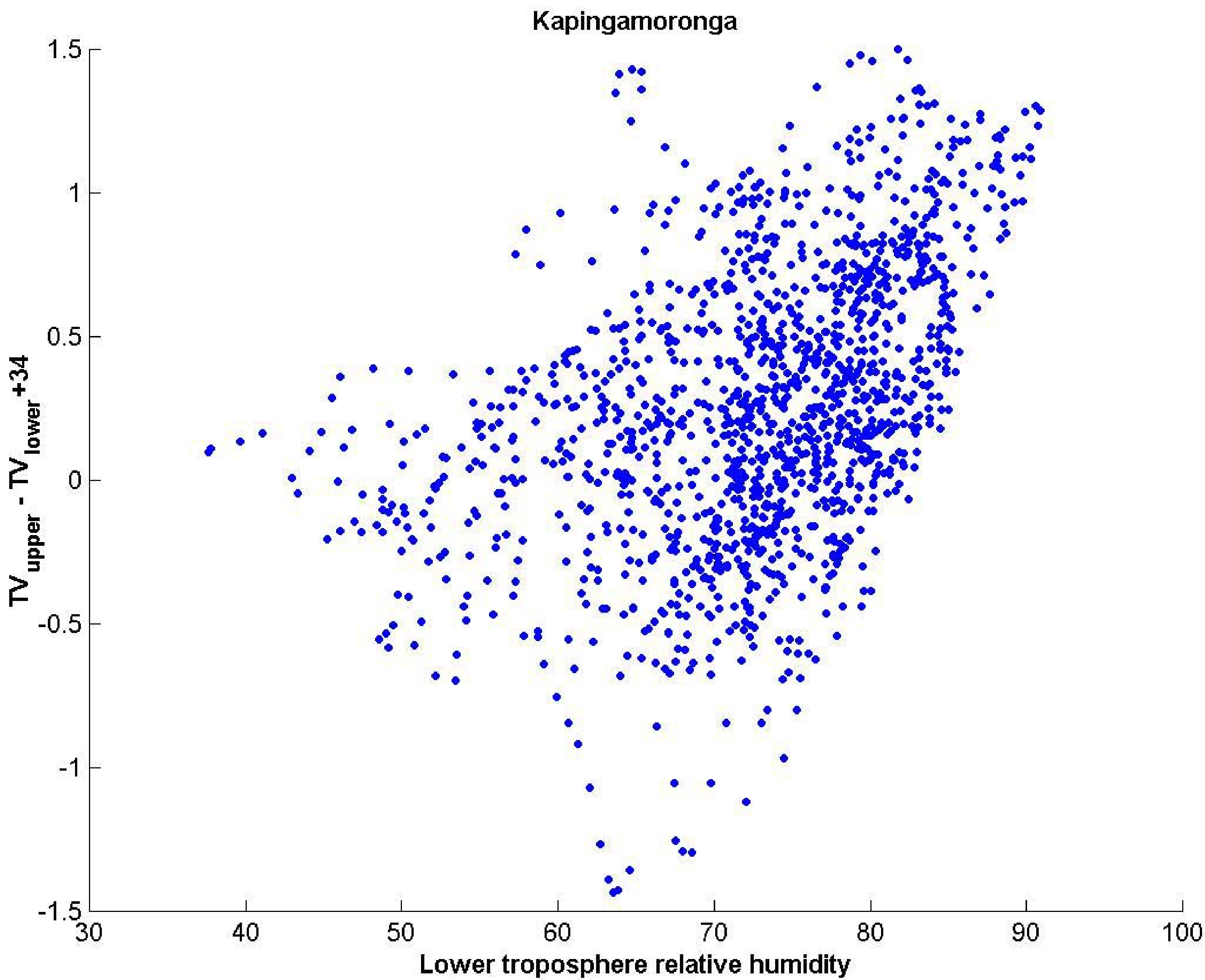


Kapingamoronga
Lower troposphere T
Upper troposphere T + 34
Density Temperature (K)
288
287.5
287
286.5
286
285.5
285
284.5
284
0
50
100
150
200
250
300
350
400
Days starting July 1



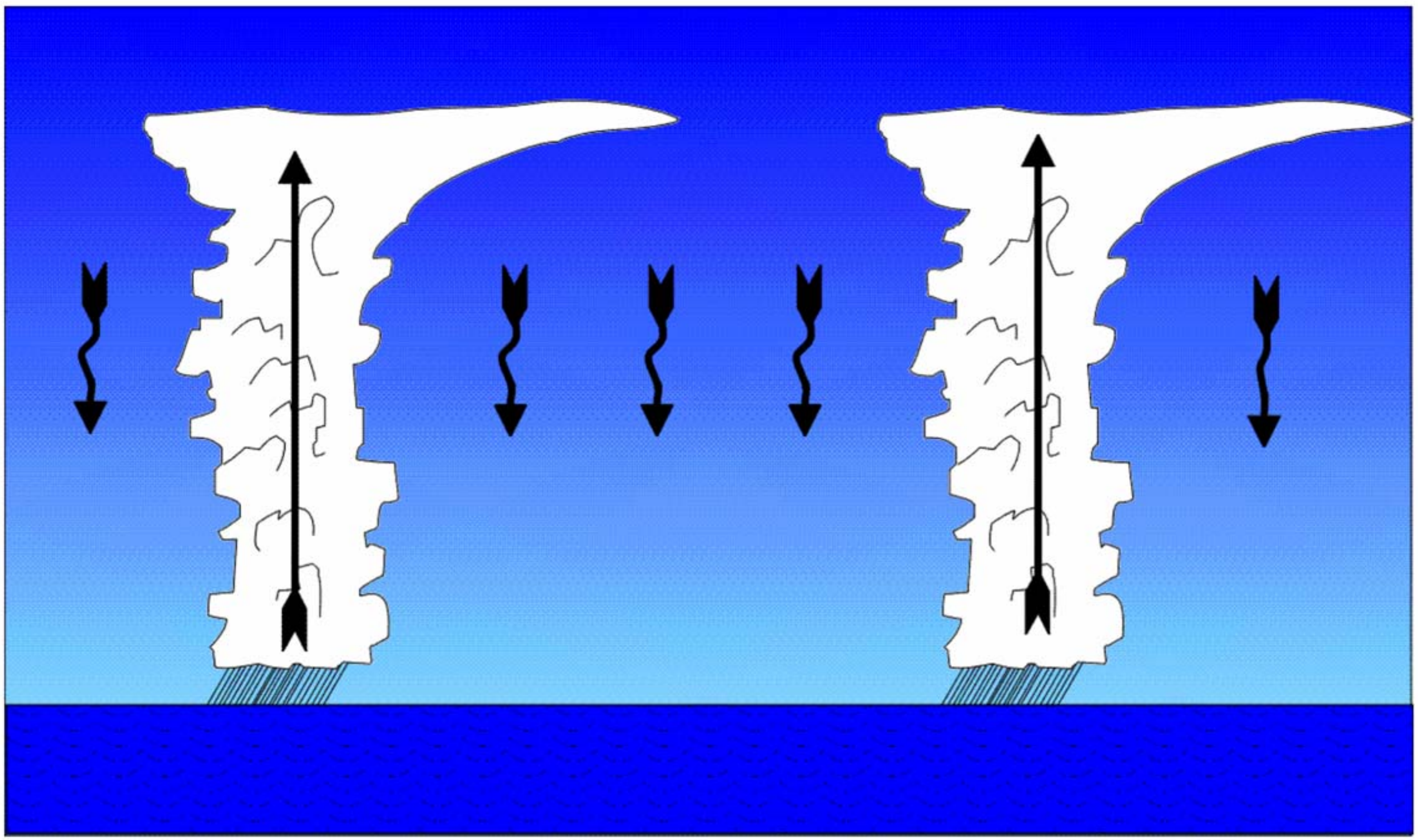
Kapingamoronga
Density Temperature (K)
Days starting July 1
TV_{upper} - TV_{lower}+34
0.1*(RH_{lower}-75)
2
1
0
-1
-2
-3
-4
0
50
100
150
200
250
300
350
400


Kapingamoronga
Density Temperature (K)
Days starting July 1
1
0.5
0
-0.5
-1
-1.5
-2
200
220
240
260
280
300
TV_{upper} - TV_{lower}+34
0.1*(RH_{lower}-75)


Kapingamoronga
1.5
1
0.5
0
-0.5
-1
-1.5
30
40
50
60
70
80
90
100
$TV_{upper} - TV_{lower} + 34$
Lower troposphere relative humidity

Radiative-Moist Convective Equilibrium

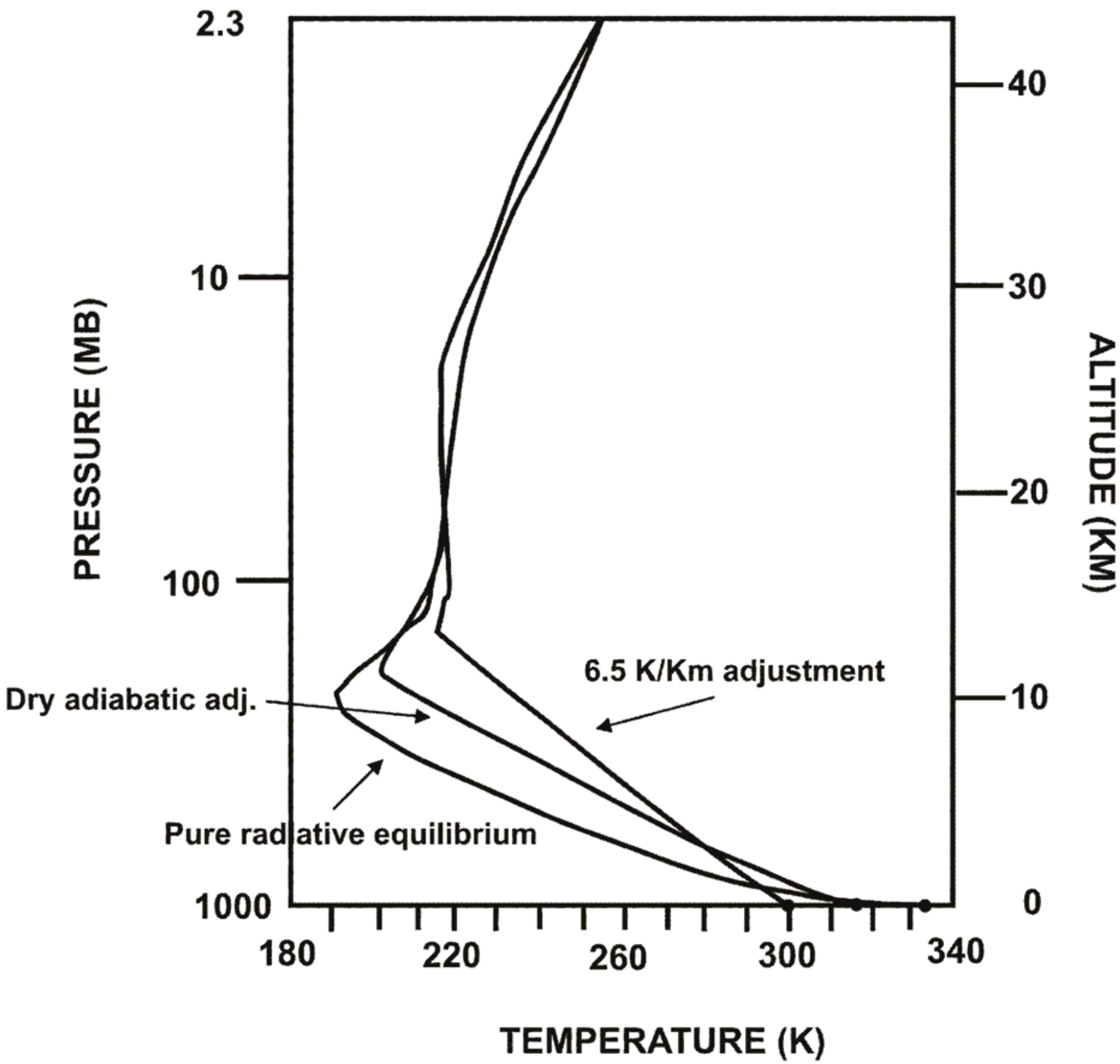
Precipitating Convection favors Widely Spaced Clouds (Bjerknes, 1938)

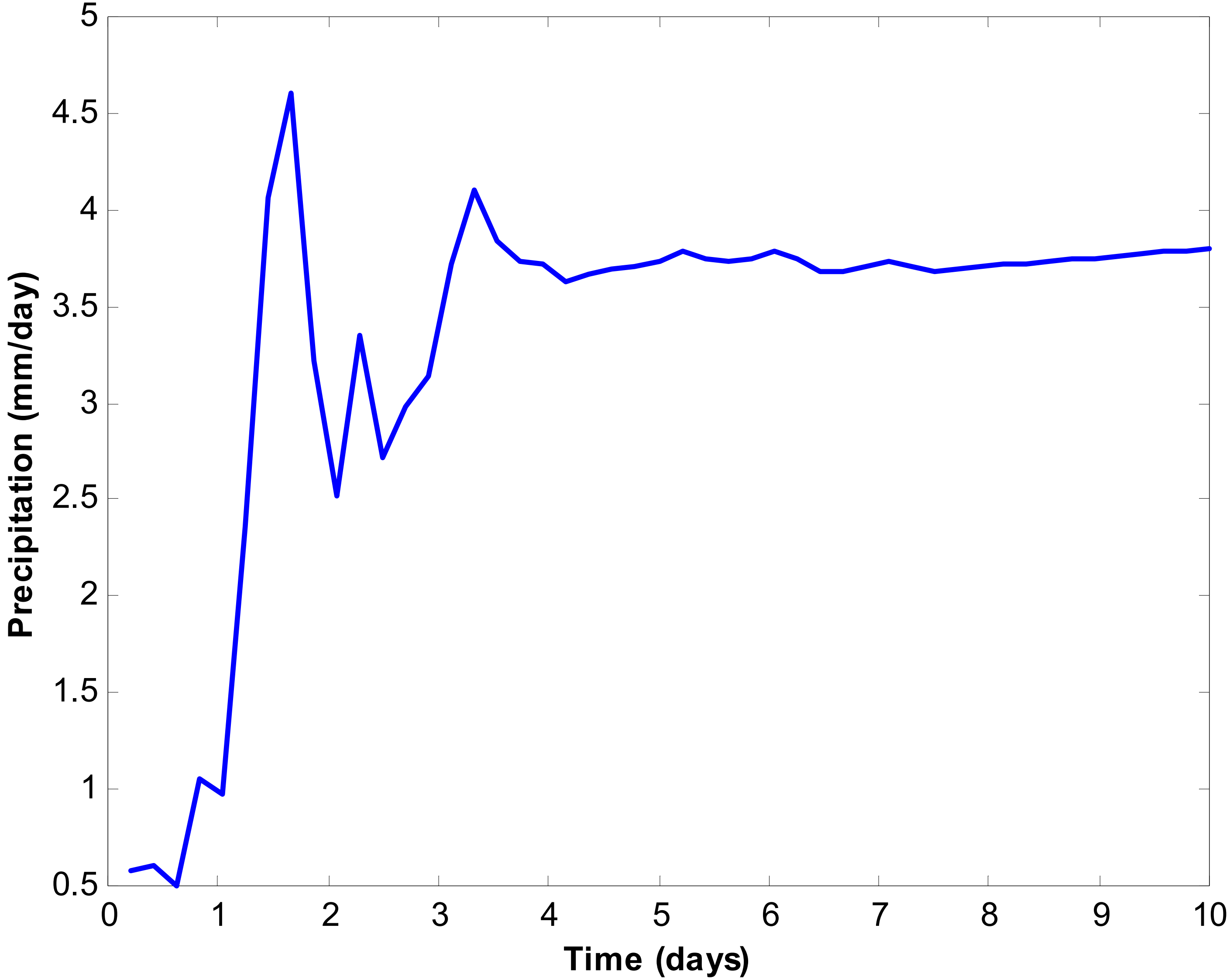


Properties:

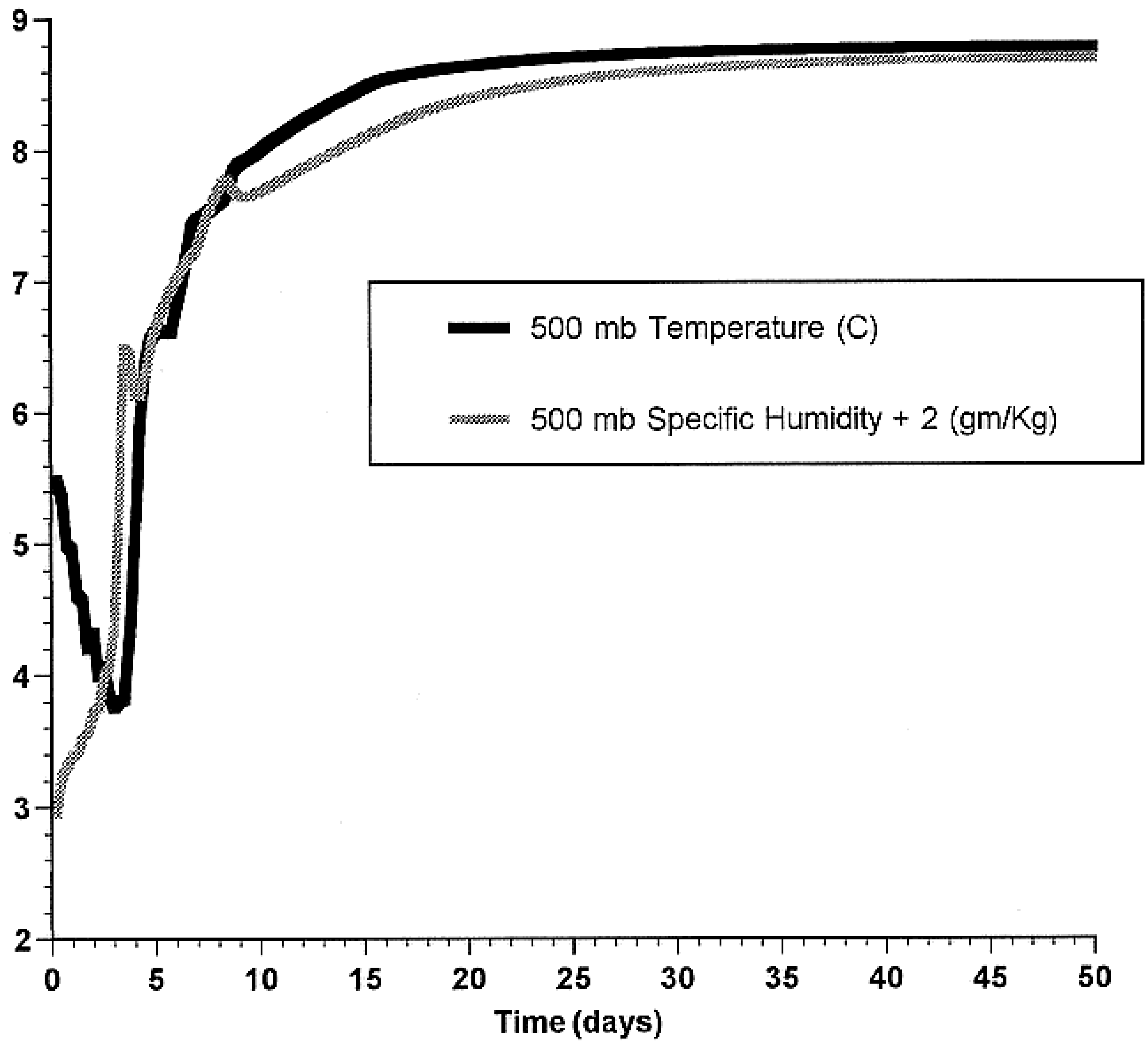
- Convective updrafts widely spaced
- Surface enthalpy flux equal to vertically integrated radiative cooling
- $$M\frac{c_p T}{\theta}\frac{\partial\theta}{\partial z} = -\dot{Q}$$
- Precipitation = Evaporation = Radiative Cooling
- Radiation and convection *highly* interactive

Manabe and Strickler 1964 calculation:

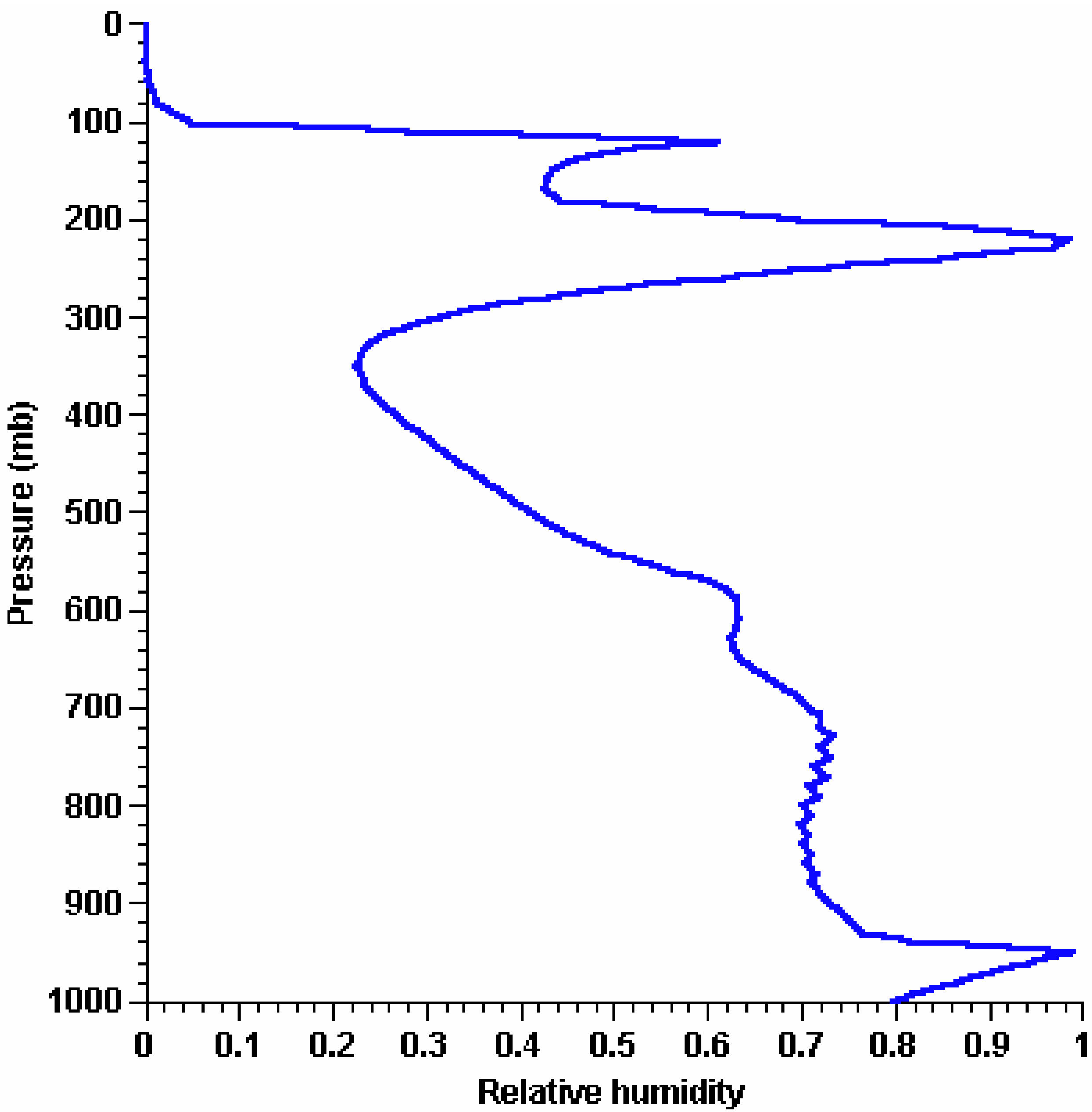




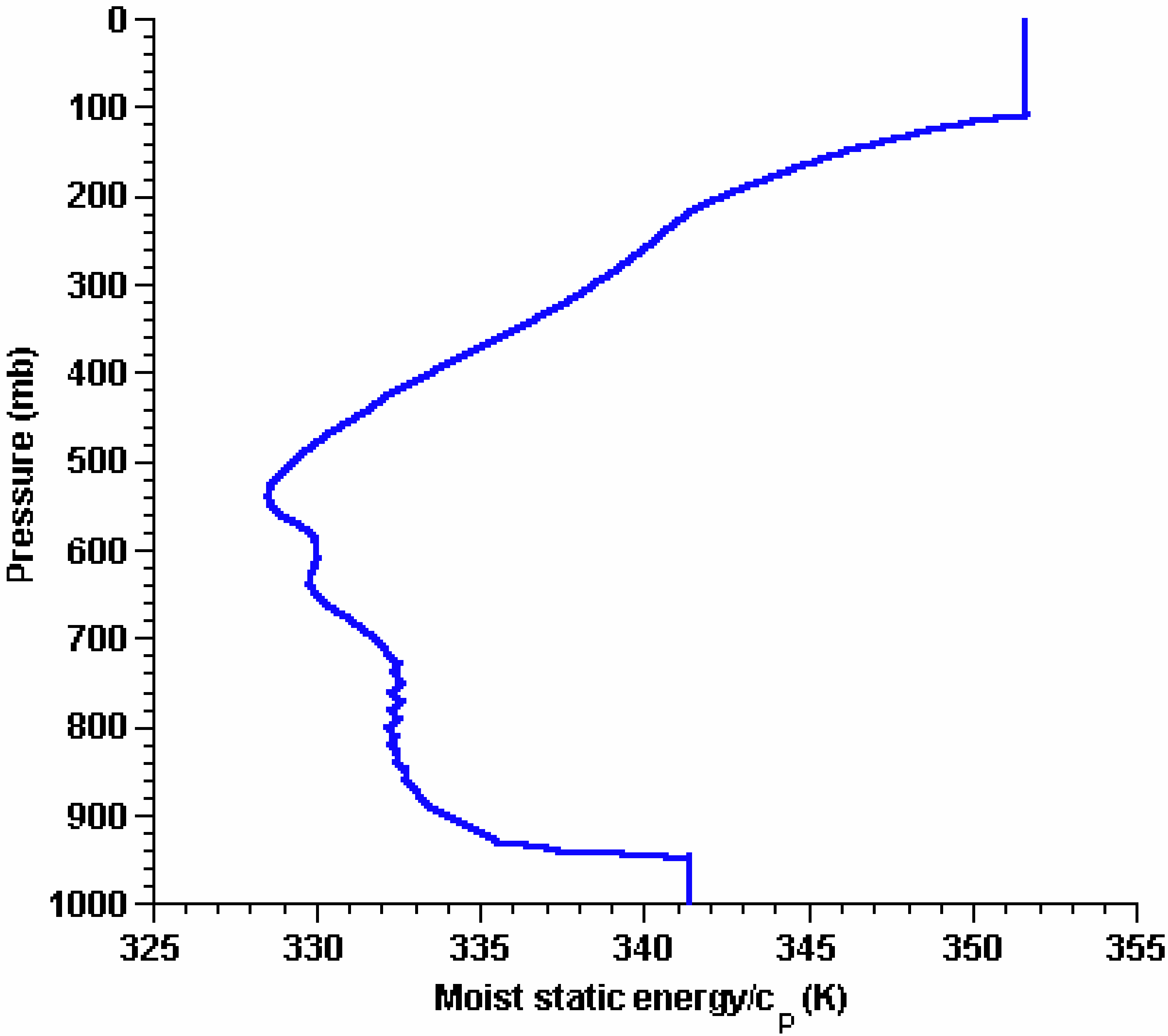
Precipitation (mm/day)
Time (days)
0
1
2
3
4
5
6
7
8
9
10
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

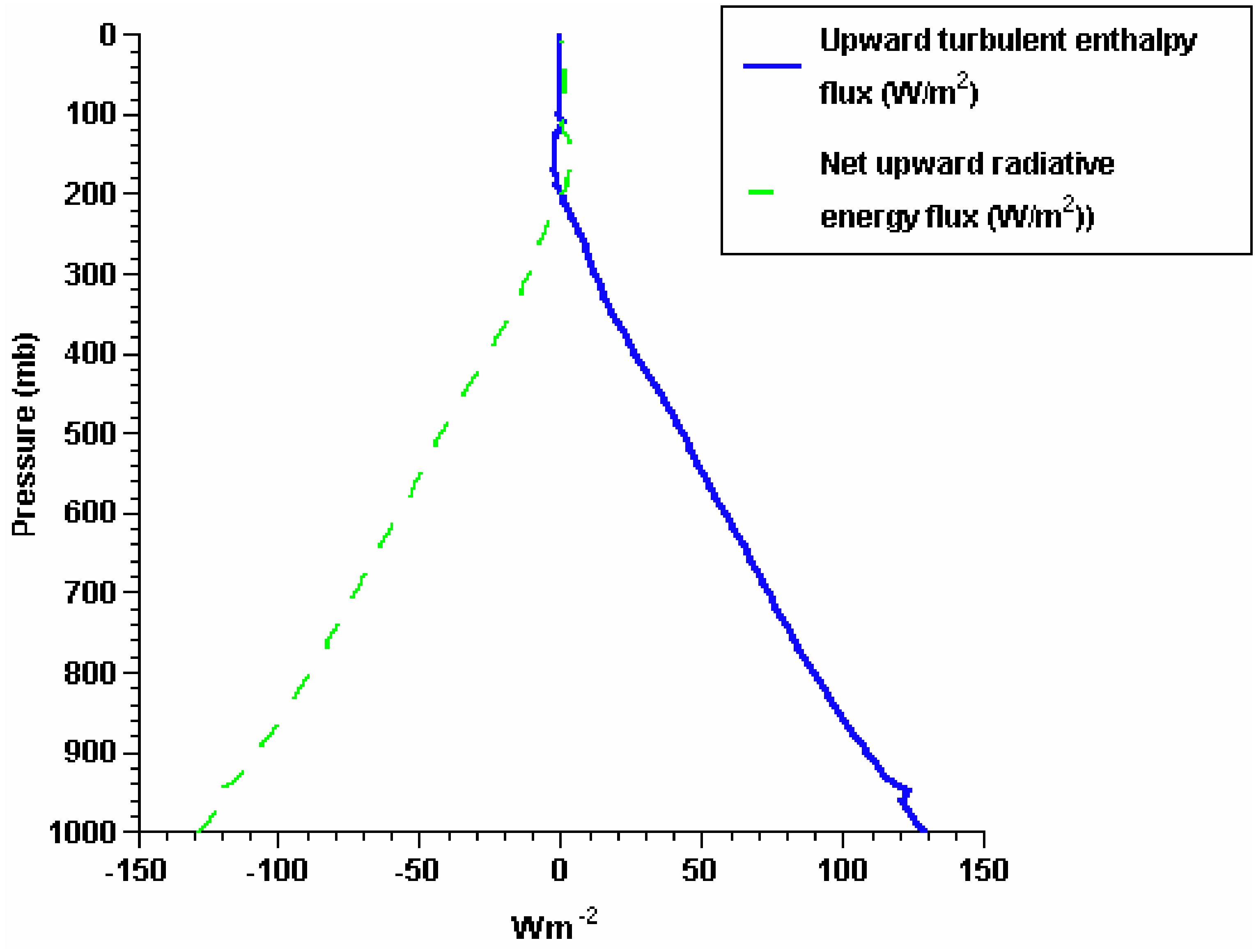


9
8
7
6
5
4
3
2
0
5
10
15
20
25
30
35
40
45
50
Time (days)
500 mb Temperature (C)
500 mb Specific Humidity + 2 (gm/Kg)

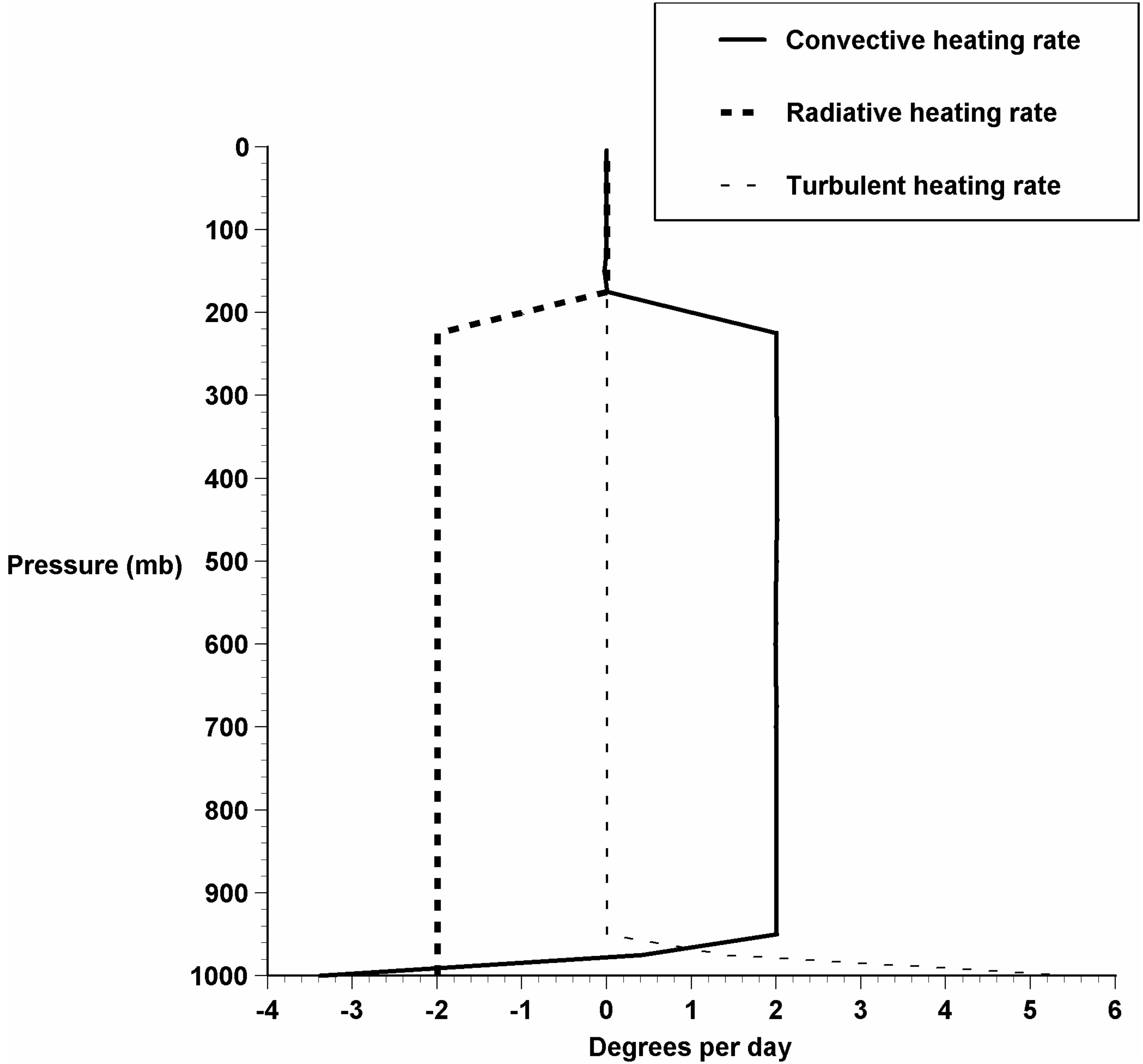


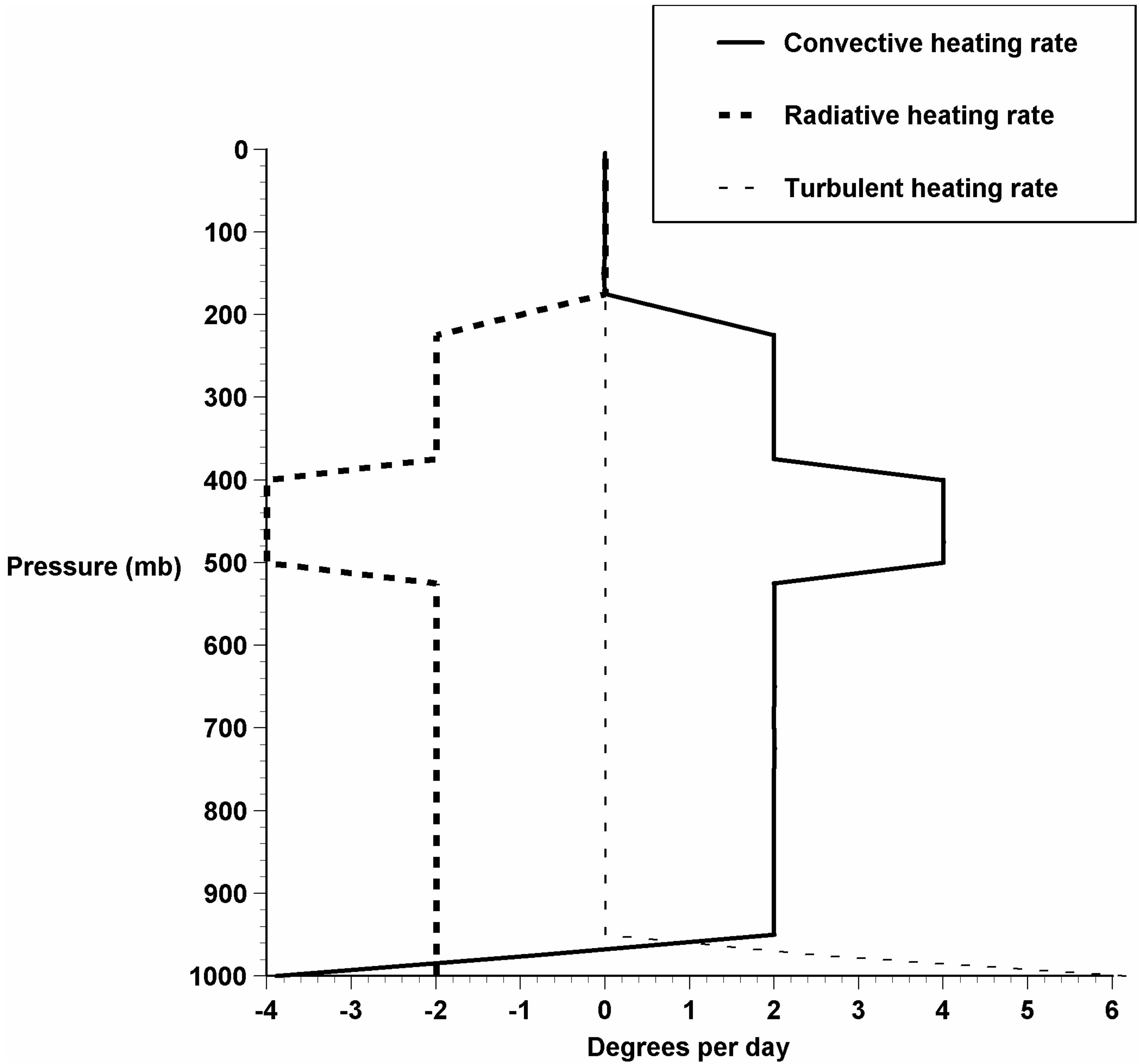
0
100
200
300
400
500
600
700
800
900
1000
Pressure (mb)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Relative humidity


Pressure (mb)
0
100
200
300
400
500
600
700
800
900
1000
325
330
335
340
345
350
355
Moist static energy/c p (K)

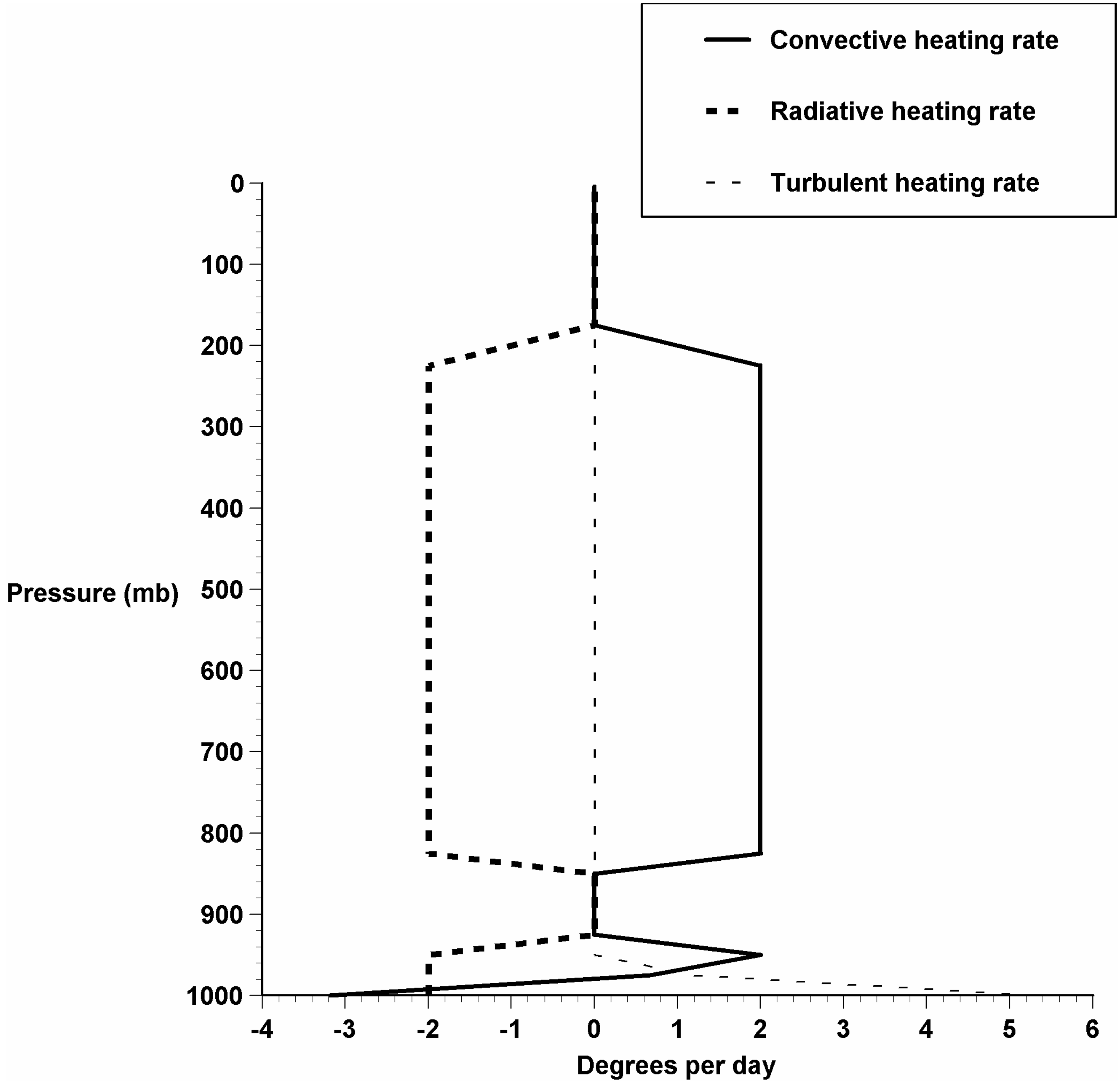


Upward turbulent enthalpy flux (W/m^2)
Net upward radiative energy flux (W/m^2))
Pressure (mb)
0
100
200
300
400
500
600
700
800
900
1000
-150
-100
-50
0
50
100
150
Wm -2

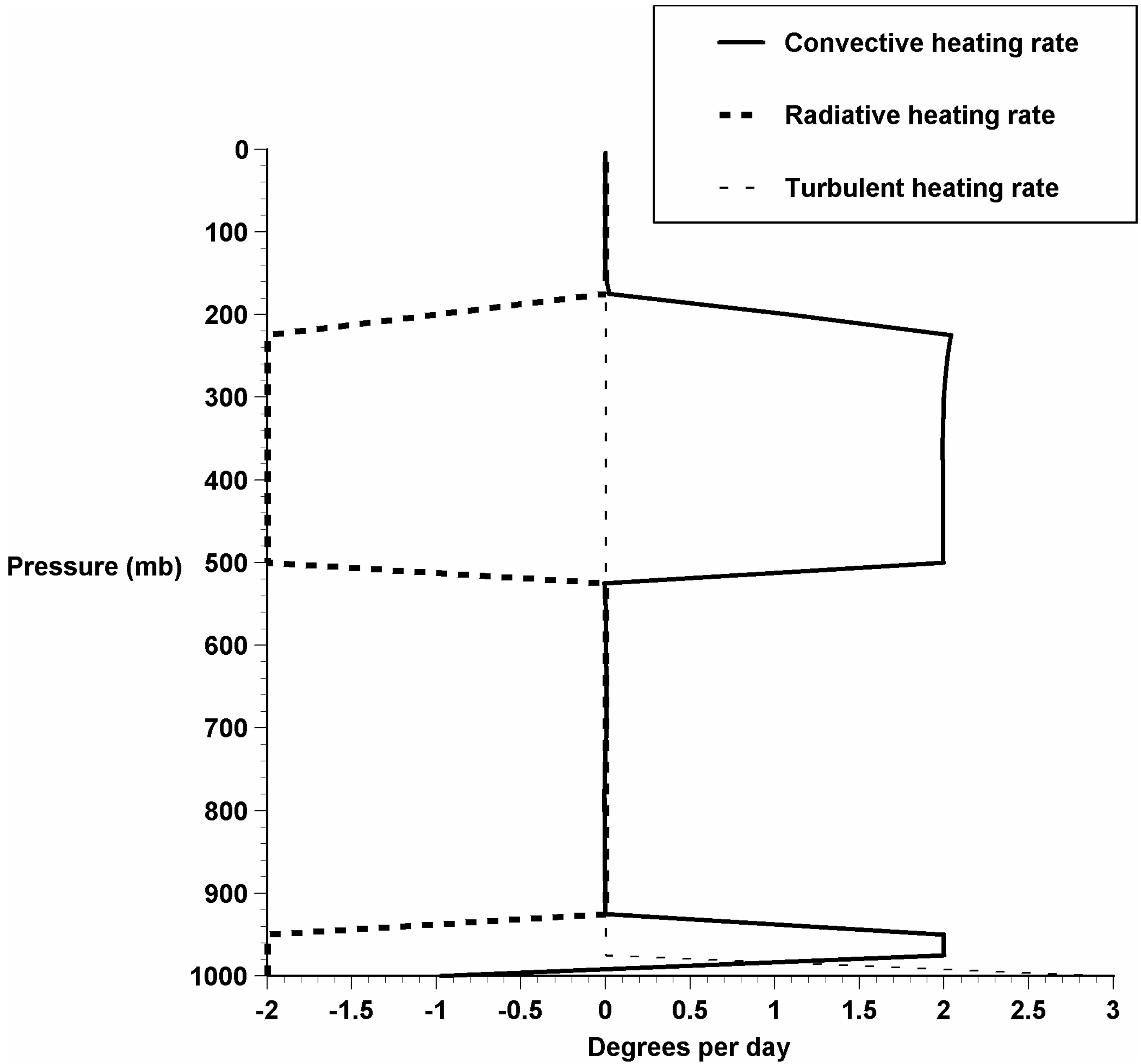

Convective heating rate
Radiative heating rate
Turbulent heating rate
Pressure (mb)
0
100
200
300
400
500
600
700
800
900
1000
-4
-3
-2
-1
0
1
2
3
4
5
6
Degrees per day



Convective heating rate
Radiative heating rate
Turbulent heating rate
Pressure (mb)
0
100
200
300
400
500
600
700
800
900
1000
-4
-3
-2
-1
0
1
2
3
4
5
6
Degrees per day

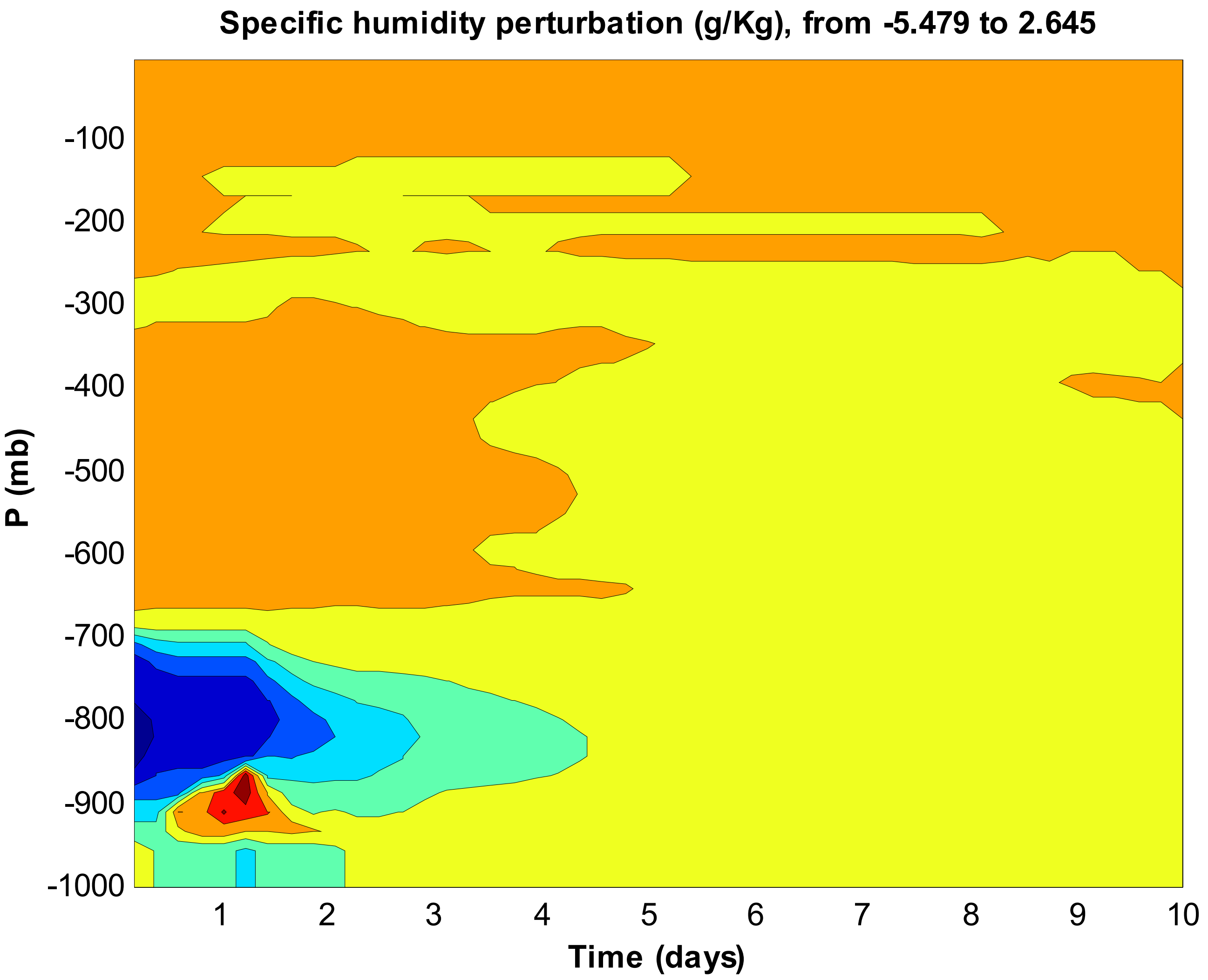


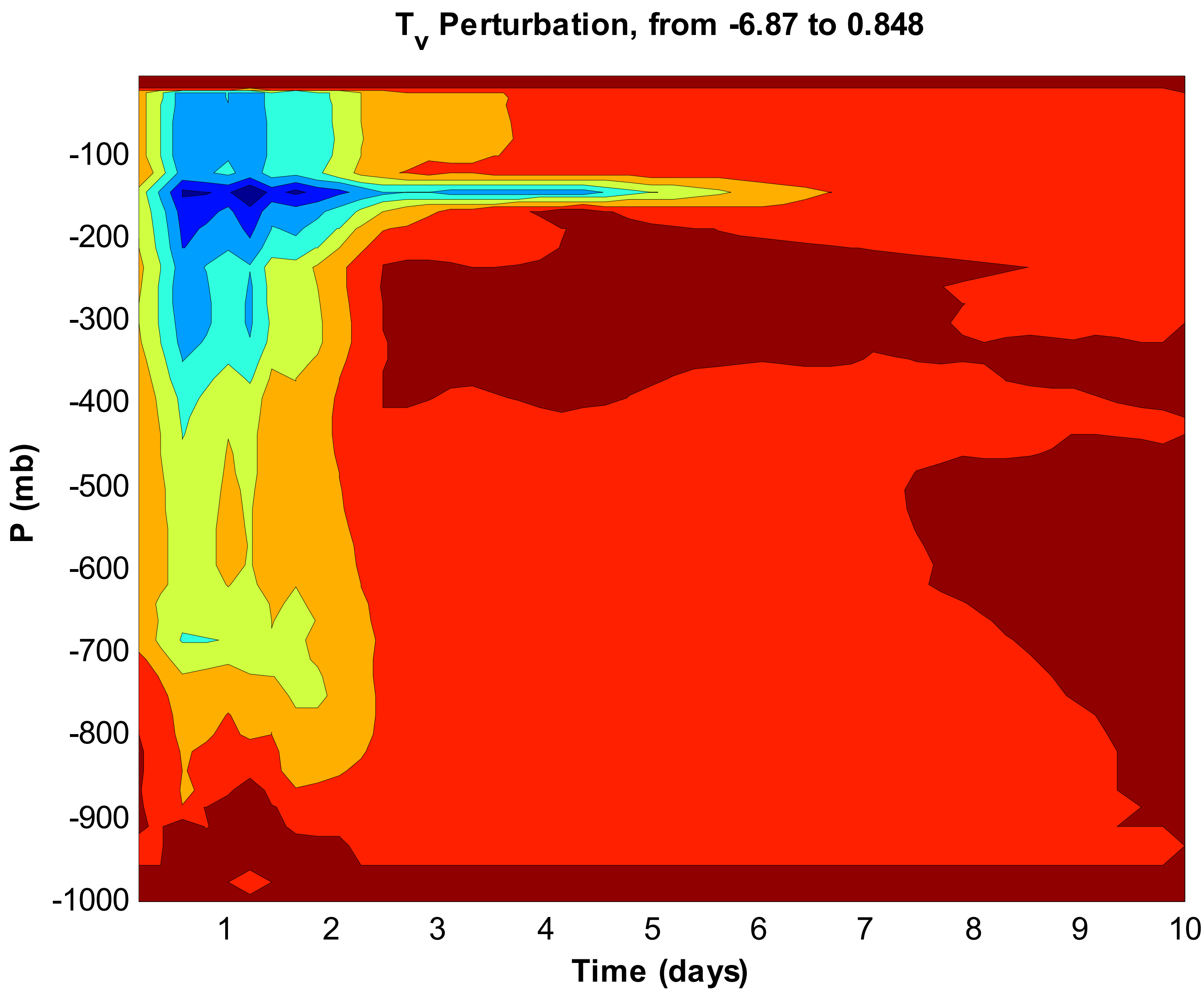
Convective heating rate
Radiative heating rate
Turbulent heating rate
Pressure (mb)
0
100
200
300
400
500
600
700
800
900
1000
-4
-3
-2
-1
0
1
2
3
4
5
6
Degrees per day



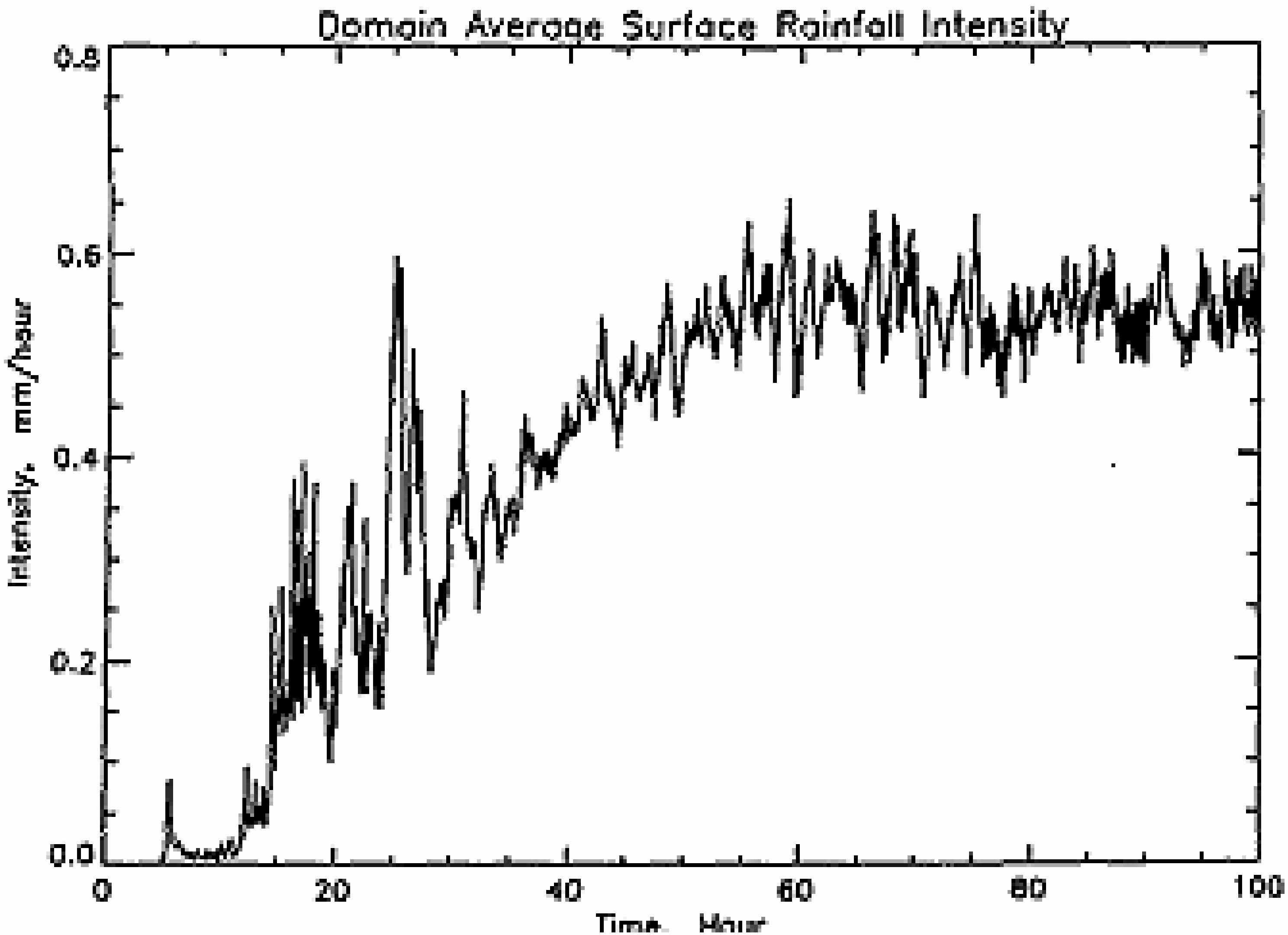
Convective heating rate
Radiative heating rate
Turbulent heating rate
Pressure (mb)
0
100
200
300
400
500
600
700
800
900
1000
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
3
Degrees per day

Recovery from mid-level specific humidity perturbation





T_v Perturbation, from -6.87 to 0.848
P (mb)
-100
-200
-300
-400
-500
-600
-700
-800
-900
-1000
1
2
3
4
5
6
7
8
9
10
Time (days)


Domain Average Surface Rainfall Intensity
0.0
0.2
0.4
0.6
0.8
Intensity, mm/hour
0
20
40
60
80
100
Time, Hour

Robe and Emanuel, J. Atmos. Sci., 1996

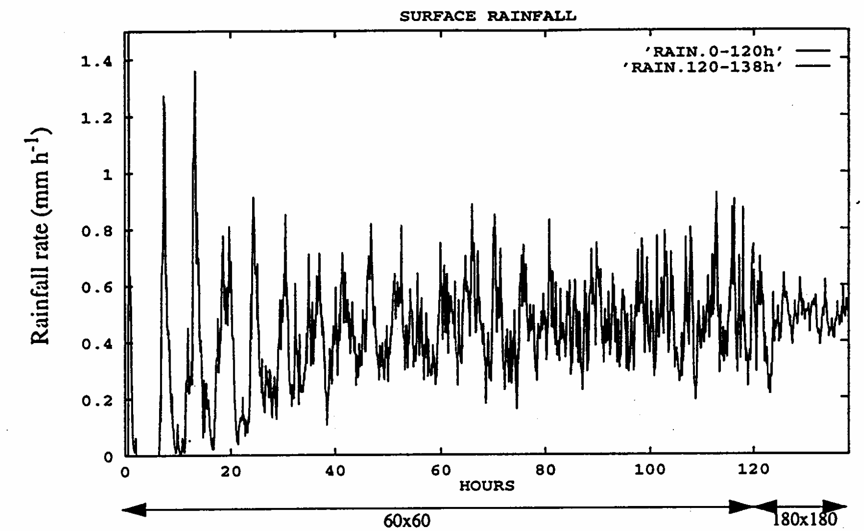
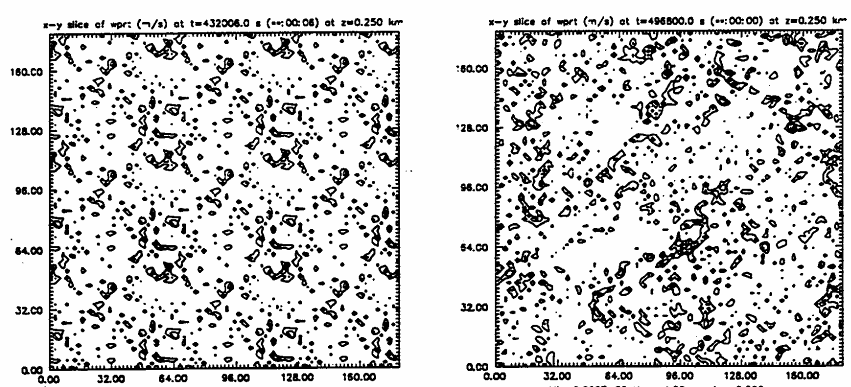
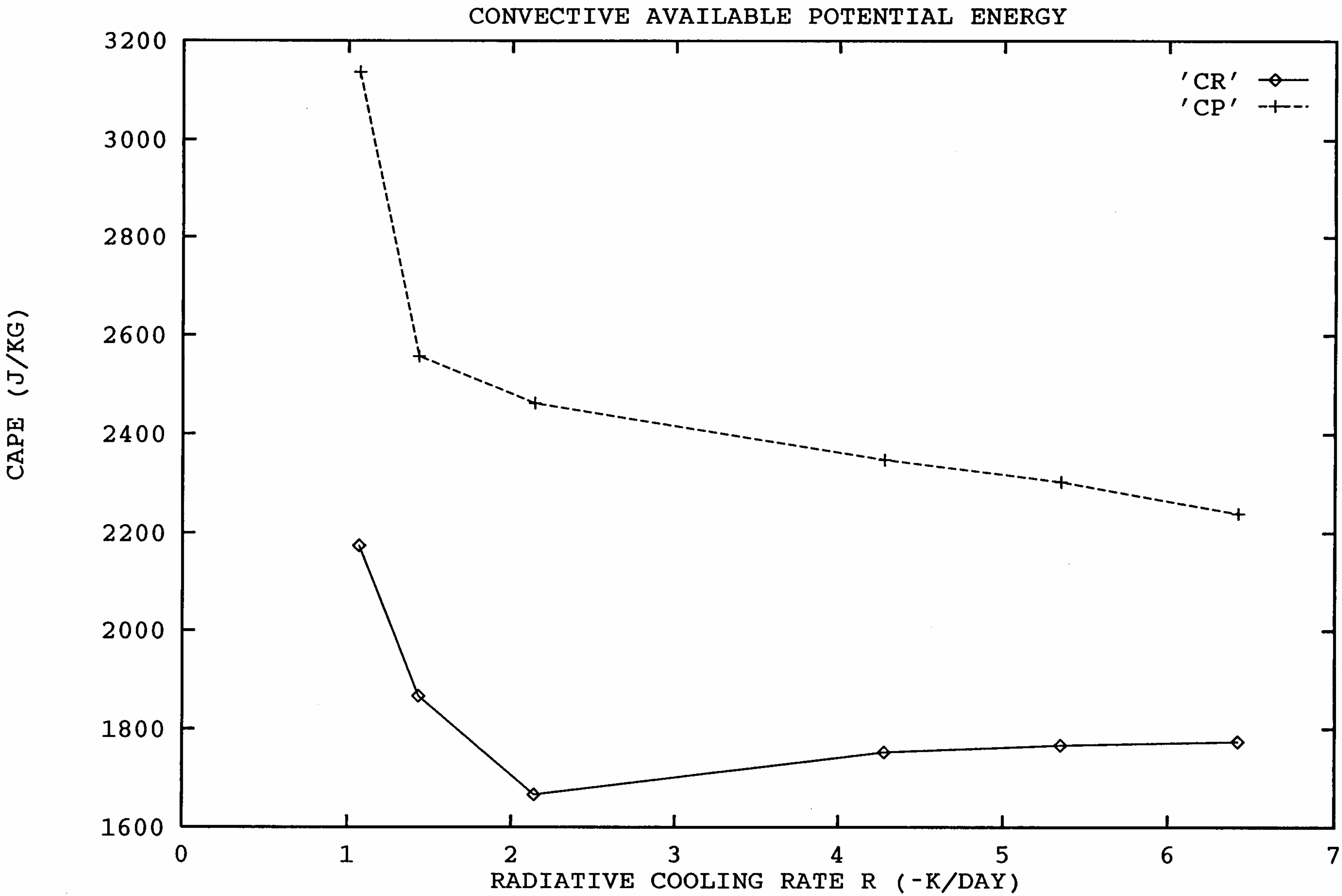


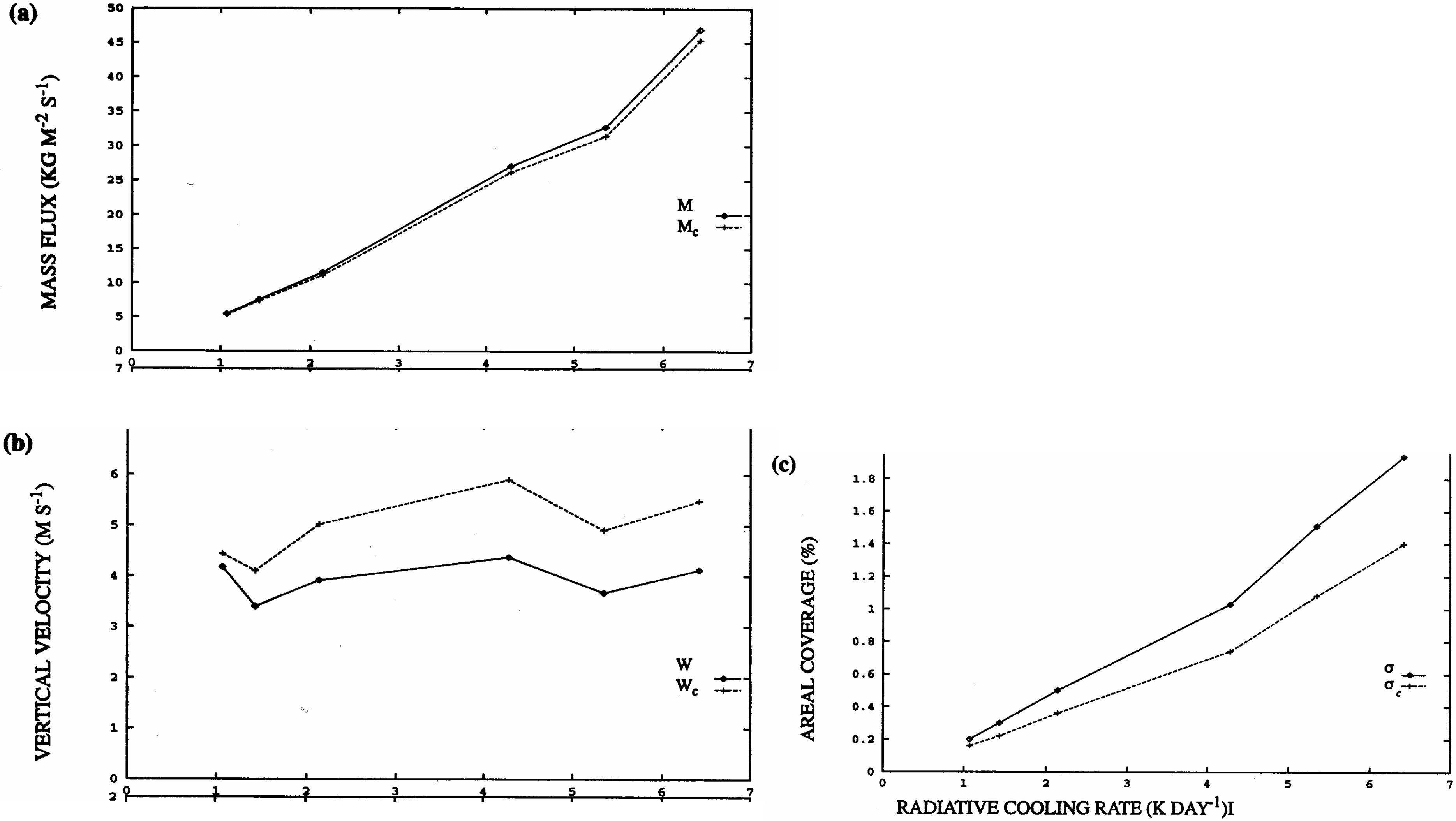
Figure 4.5: time-series of the horizontally averaged rainfall at the ground for R= -5.4 K/day. The domain extends over 60 x 60 km^2 for the first 120 hours, and over 180 x 180 km^2 for the last 18 hours.



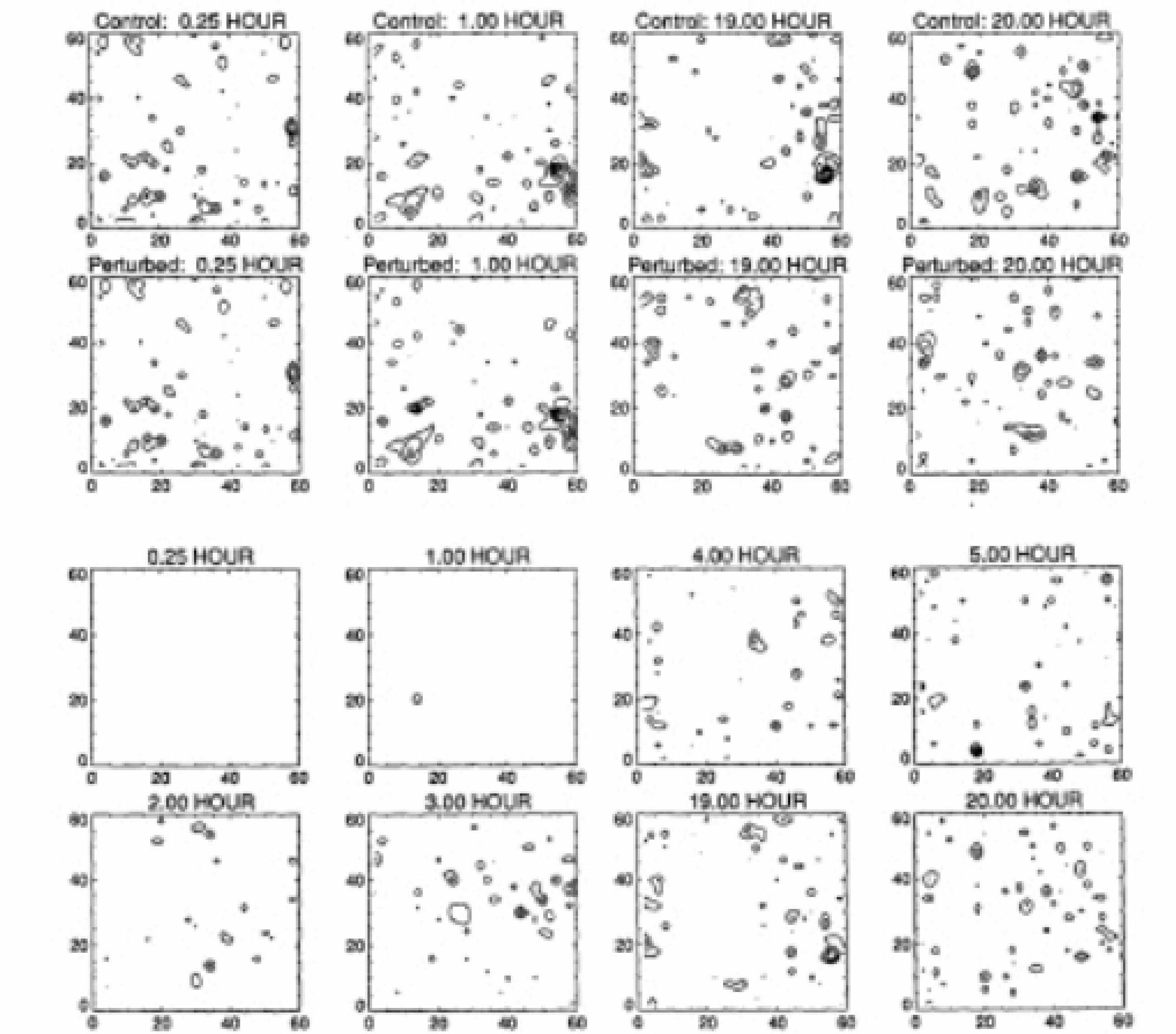
120 h　　138 h

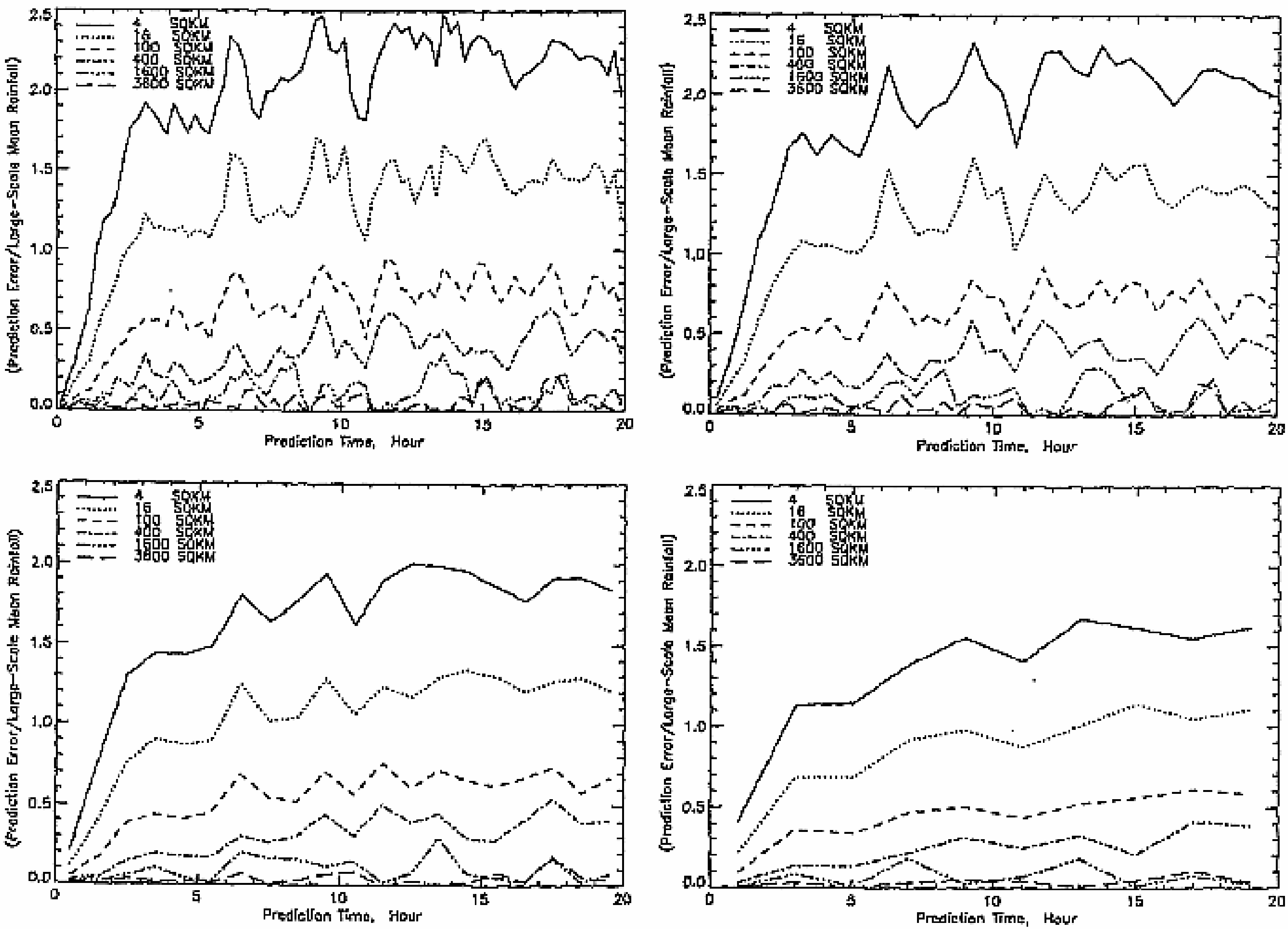


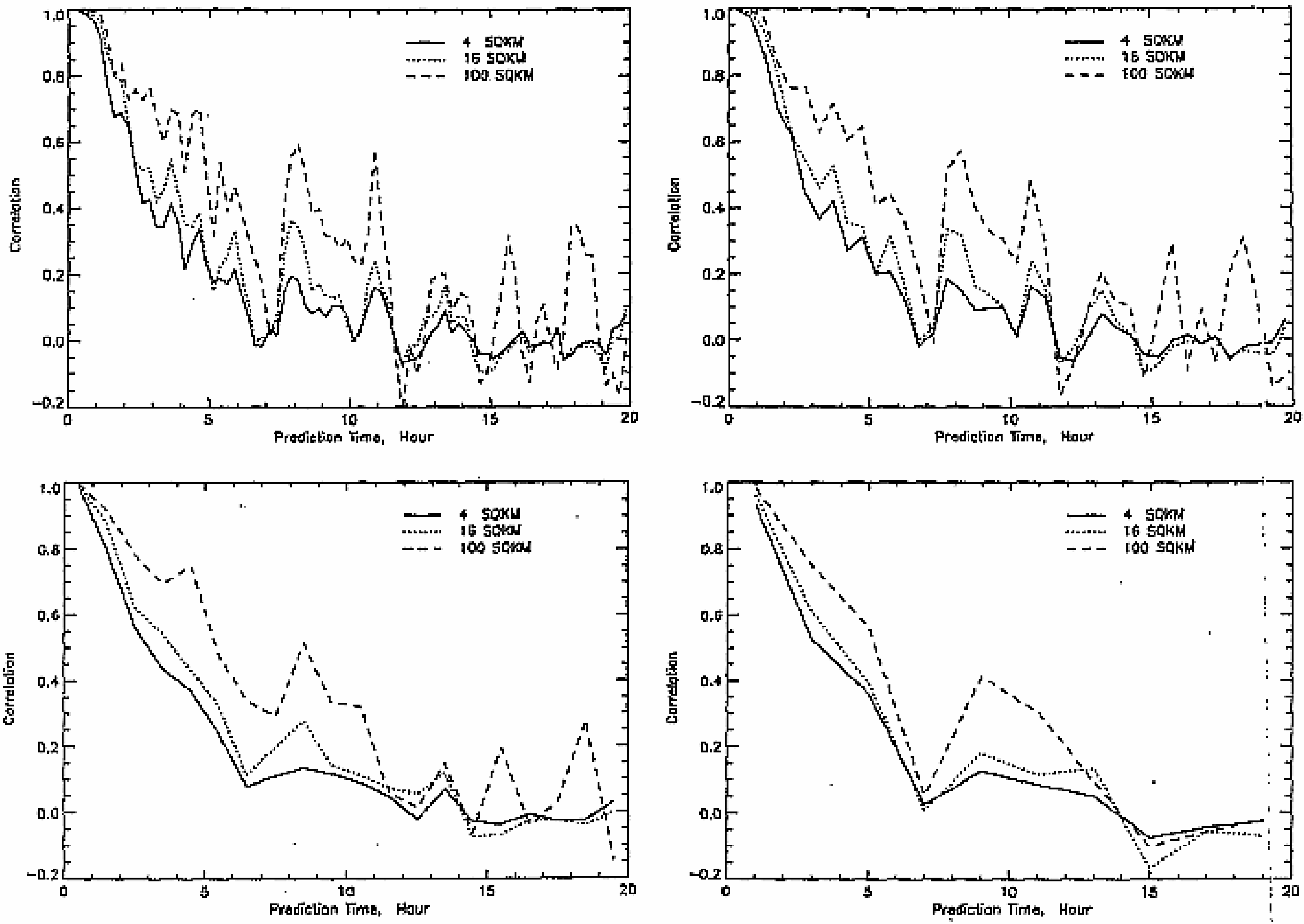
CONVECTIVE AVAILABLE POTENTIAL ENERGY
CAPE (J/KG)
3200
3000
2800
2600
2400
2200
2000
1800
1600
0
1
2
3
4
5
6
7
RADIATIVE COOLING RATE R (-K/DAY)
'CR'
'CP'


(a)
MASS FLUX (KG M-2 S-1)
M
Mc
(b)
VERTICAL VELOCITY (M S-1)
W
Wc
(c)
AREAL COVERAGE (%)
σ
σc
RADIATIVE COOLING RATE (K DAY-1)I

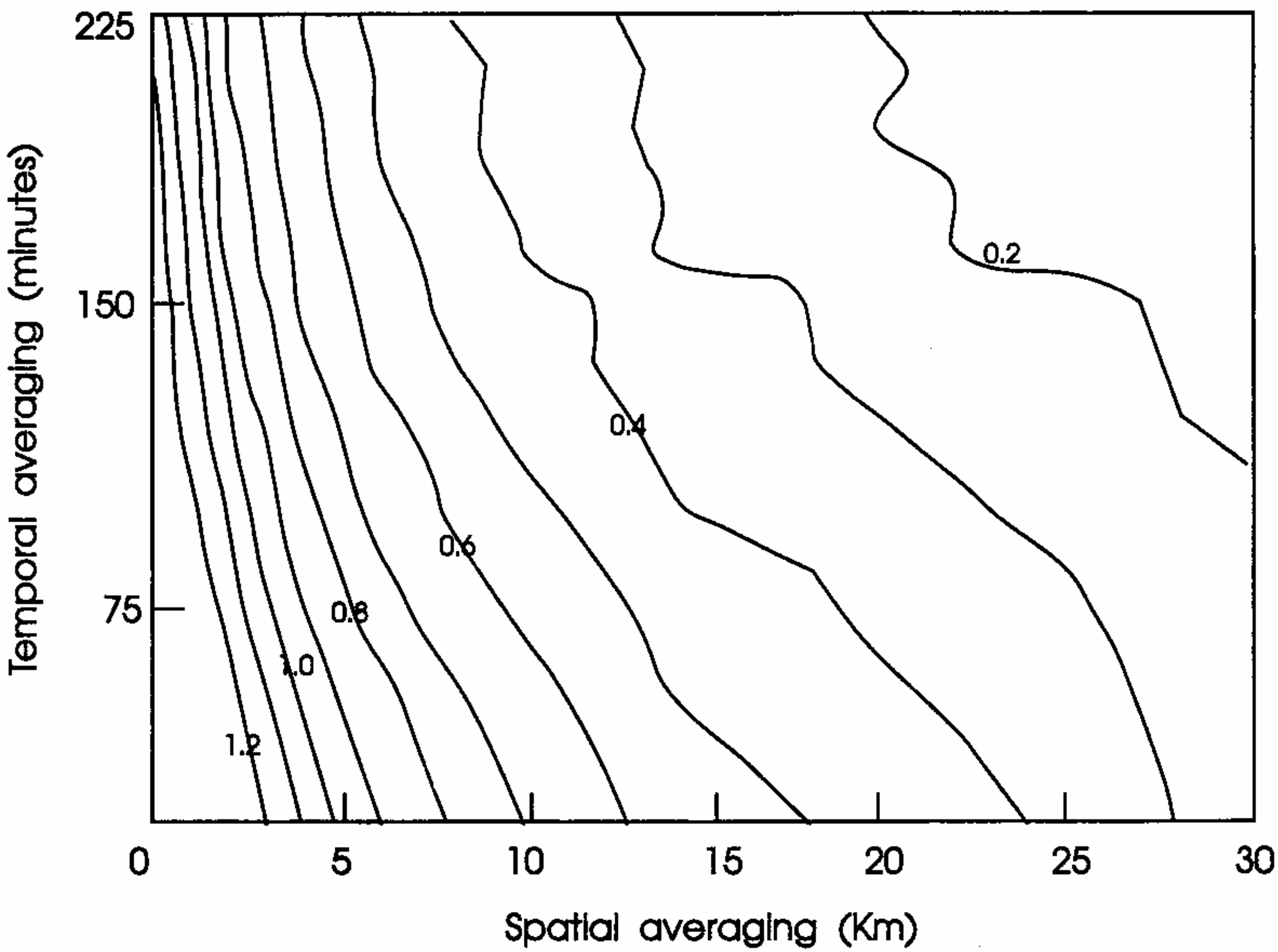
Islam et al. Predictability Experiments




4 SQKM
16 SQKM
100 SQKM
400 SQKM
1600 SQKM
3600 SQKM
(Prediction Error/Large-Scale Mean Rainfall)
Prediction Time, Hour
4 SQKM
16 SQKM
100 SQKM
400 SQKM
1600 SQKM
3600 SQKM
(Prediction Error/Large-Scale Mean Rainfall)
Prediction Time, Hour
4 SQKM
16 SQKM
100 SQKM
400 SQKM
1600 SQKM
3600 SQKM
(Prediction Error/Large-Scale Mean Rainfall)
Prediction Time, Hour
4 SQKM
16 SQKM
100 SQKM
400 SQKM
1600 SQKM
3600 SQKM
(Prediction Error/Large-Scale Mean Rainfall)
Prediction Time, Hour



4 SQKM
16 SQKM
100 SQKM
Correlation
Prediction Time, Hour
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0
5
10
15
20


225
150
75
Temporal averaging (minutes)
0
5
10
15
20
25
30
Spatial averaging (Km)
0.2
0.4
0.6
0.8
1.0
1.2