Above a thin boundary layer, most atmospheric convection involved phase change of water: Moist Convection
Moist Convection

• Significant heating owing to phase changes of water
• Redistribution of water vapor – most important greenhouse gas
• Significant contributor to stratiform cloudiness – albedo and longwave trapping
Water Variables

Mass concentration of water vapor (specific humidity):

\[ q \equiv \frac{M_{H_2O}}{M_{air}} \]

Vapor pressure (partial pressure of water vapor): \( e \)

Saturation vapor pressure: \( e^* \)

C-C: \[ e^* = 6.112 \, hPa \, e \frac{17.67(T-273)}{T+30} \]

Relative Humidity: \[ \mathcal{H} \equiv \frac{e}{e^*} \]
The Saturation Specific Humidity

Ideal Gas Law:

\[ p = \rho \frac{R \ast T}{\bar{m}} \]

\[ e = \rho_v \frac{R \ast T}{m_v} \]

\[ q = \rho_v \rho \frac{m_v e}{\bar{m} \bar{p}} \]

\[ q^* = \frac{m_v e^*}{\bar{m} \bar{p}} \]
Phase Equilibria

- Ice
- Liquid
- Critical Point
- Triple Point
- Vapor

Vapor Pressure

Temperature
Bringing Air to Saturation

\[ e = q \frac{\bar{m}}{m_v} \]

\[ e^* = e^*(T) \]

1. Increase \( q \) (or \( p \))

2. Decrease \( e^* (T) \)
When Saturation Occurs...

- Heterogeneous Nucleation
- Supersaturations very small in atmosphere
- Drop size distribution sensitive to size distribution of cloud condensation nuclei
Ice Nucleation Problematic

![Graph showing the relationship between cloud top temperature and the percentage of clouds with ice particle concentrations above detectable levels.](image-url)
Precipitation Formation:

- Stochastic coalescence (sensitive to drop size distributions)
- Bergeron-Findeisen Process
- Strongly nonlinear function of cloud water concentration
- Time scale of precipitation formation ~10-30 minutes
Stability

No simple criterion based on entropy:

\[ s_d = c_p \ln \left( \frac{T}{T_0} \right) - R_d \ln \left( \frac{p}{p_0} \right) \]

\[ \alpha = \alpha (s_d, p) \]

\[ s = c_p \ln \left( \frac{T}{T_0} \right) - R_d \ln \left( \frac{p}{p_0} \right) + L_v \frac{q}{T} - qR_v \ln (\mathcal{H}) \]

\[ \alpha = \alpha (s, p, q_t) \]
Virtual Temperature and Density Temperature

Assume all condensed water falls at terminal velocity

\[ \alpha = \frac{V_a + V_c}{M_d + M_v + M_c} \]

\[ pV = nRT \]

\[ V_a = \frac{R^*T}{p} \left( \frac{M_d}{m_d} + \frac{M_v}{m_v} \right), \]
\[ m_d \equiv \frac{1}{\frac{1}{M_d} \sum_i \frac{M_i}{m_i}} \]

\[ \rightarrow V_a = \frac{R_d T}{p} \left( M_d + \frac{M_v}{\epsilon} \right), \]

where

\[ \epsilon \equiv \frac{m_v}{m_d} \approx 0.622 \]

\[ R_d \equiv \frac{R^*}{m_d} \]
\[ \alpha = \frac{V_a + V_c}{M_d + M_v + M_c} = \frac{R_d T}{p} \left( 1 - q_t + \frac{q}{\varepsilon} \right) \left( 1 + \frac{q_c}{1-q_c} \frac{\rho_a}{\rho_c} \right) \]

\[ \cong \frac{R_d T}{p} \left( 1 - q_t + \frac{q}{\varepsilon} \right) \]

\[ q_t \equiv \frac{M_v + M_c}{M}, \quad q \equiv \frac{M_v}{M} \]

Density temperature:

\[ T_\rho \equiv T \left( 1 - q_t + \frac{q}{\varepsilon} \right) \]

\[ \alpha = \frac{R_d T_\rho}{p} \]
Trick:

Define a saturation entropy, $s^*$:

$$s^* \equiv s(T, p, q^*)$$

$$\alpha = \alpha(s^*, p, q_t)$$

We can add an arbitrary function of $q_t$ to $s^*$ such that

$$\alpha \approx \alpha(s^*, p)$$
Stability Assessment using Tephigrams:
Stability Assessment using Tephigrams:

Convective Available Potential Energy (CAPE):

\[ CAPE_i = \int_{p_n}^{p_i} (\alpha_p - \alpha_e) dp = \int_p^{p_i} R_d \left( T_{\rho_p} - T_{\rho_e} \right) d \ln(p) \]
Other Stability Diagrams:
“Air-Mass” Showers:

- **Towering Cumulus Stage**
- **Mature Stage**
- **Dissipation Stage**
Fig. 15. Radar echo, plane paths, measured draft data, and cell outlines, 1310 EST 9 July 1946.

Fig. 16. Radar echo, plane paths, measured draft data, and cell outlines, 1320 EST 9 July 1946.
Tropical Soundings

November - February

- $T_v\text{ av}$
- $T_{vp}\text{ av}$
Annual Mean Kapingamoronga:
Radiative-Moist Convective Equilibrium
Precipitating Convection favors Widely Spaced Clouds (Bjerknes, 1938)
Properties:

- Convective updrafts widely spaced
- Surface enthalpy flux equal to vertically integrated radiative cooling
  \[ M \frac{c_p T}{\theta} \frac{\partial \theta}{\partial z} = -\dot{Q} \]
- Precipitation = Evaporation = Radiative Cooling
- Radiation and convection *highly* interactive
Manabe and Strickler 1964 calculation:
Recovery from mid-level specific humidity perturbation

Specific humidity perturbation (g/Kg), from -5.479 to 2.645
$T_v$ Perturbation, from -6.87 to 0.848
Figure 4.5: time-series of the horizontally averaged rainfall at the ground for R = -5.4 K/day. The domain extends over 60 x 60 km$^2$ for the first 120 hours, and over 180 x 180 km$^2$ for the last 18 hours.
Islam et al. Predictability Experiments