Dynamics of the Zonal-Mean, Time-Mean Tropical Circulation

First consider a hypothetical planet like Earth, but with no continents and no seasons and for which the only friction acting on the atmosphere is at the surface.

This planet has an exact nonlinear equilibrium solution for the flow of the atmosphere, characterized by

1. Every column is in radiative-convective equilibrium,

2. Wind vanishes at planet’s surface

3. Horizontal pressure gradients balanced by Coriolis accelerations

Hydrostatic balance:

$$\frac{\partial p}{\partial z} = -\rho g$$

In pressure coordinates:

$$\frac{\partial \phi}{\partial p} = -\alpha,$$

$$where$$

$$\alpha = {}^{1}\!/_{\rho} \equiv specific\ volume,$$

$$\phi = gz \equiv geopotential$$

Geostrophic balance on a sphere:

$$\left(\frac{\partial \phi}{a\partial \theta}\right)_p = -2\Omega \sin\theta u_{rel} - \frac{u_{rel}^2}{a}\tan\theta$$

Can be phrased in terms of absolute angular momentum per unit mass:

$$M = a\cos\theta\left(\Omega a\cos\theta + u\right)$$

$$\rightarrow \quad \frac{\partial \phi}{\partial \theta} = -\sin\theta\left[\frac{M^2 - \Omega^2 a^4 \cos^4\theta}{a^2\cos^3\theta}\right]$$

Eliminate ϕ using hydrostatic equation:

$$\frac{1}{a^2}\frac{\tan\theta}{\cos^2\theta}\frac{\partial M^2}{\partial p}=\left(\frac{\partial \alpha}{\partial \theta}\right)_p=\left(\frac{\partial T}{\partial p}\right)_{s^*}\left(\frac{\partial s^*}{\partial \theta}\right)_p$$

(Thermal wind equation)

Take $s^* = s^*(\theta)$ (convective neutrality)

Integrate upward from surface ($p=p_0$), taking u=0 at surface:

$$M^2 = a^2\cos^2\theta\left[\Omega^2 a^2\cos^2\theta - \cot\theta\left(T_s - T\right)\frac{\partial s^*}{\partial\theta}\right]$$

$$\rightarrow \quad u^2 + 2\Omega a\cos\theta\, u + \cot\theta\left(T_s - T\right)\frac{\partial s^*}{\partial\theta} = 0.$$

$$\boxed{u \cong -\frac{\left(T_s - T\right)}{2\Omega\sin\theta}\frac{\partial s^*}{\partial y}}$$

Implies strongest west-east winds where entropy gradient is strongest, weighted toward equator

Two potential problems with this solution:

1. Not enough angular momentum available for required west-east wind,

2. Equilibrium solution may be unstable

How much angular momentum is needed?

At the tropopause,

$$M^2 = a^2 \cos^2 \theta \left[\Omega^2 a^2 \cos^2 \theta - \cot \theta \left(T_s - T_t \right) \frac{\partial s^*}{\partial \theta} \right]$$

How much angular momentum is available?

$$M_{eq}^2 = \Omega^2 a^4$$

$$\rightarrow \quad \left(T_s - T_t \right) \frac{\partial s^*}{\partial \theta} \geq - \frac{\Omega^2 a^2 \tan \theta \left(1 - \cos^4 \theta \right)}{\cos^2 \theta}$$

More generally, we require that angular momentum decrease away from the equator:

$$\frac{1}{\sin\theta}\frac{\partial M^2}{\partial\theta} < 0$$

$$\rightarrow \quad 4\Omega^2 a^2 \cos^3\theta + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left[\cos^2\theta \cot\theta \left(T_s - T_t\right)\frac{\partial s^*}{\partial\theta}\right] > 0.$$

Let $\quad y \equiv \sec^2\theta$

$$\rightarrow \quad 1 + y^3 \frac{\partial}{\partial y}\left(\frac{T_s - T_t}{\Omega^2 a^2}\frac{\partial s^*}{\partial y}\right) > 0$$

What happens when this condition is violated?

Overturning circulation must develop

Acts to drive actual entropy distribution back toward criticality

Postulate constant angular momentum at tropopause:

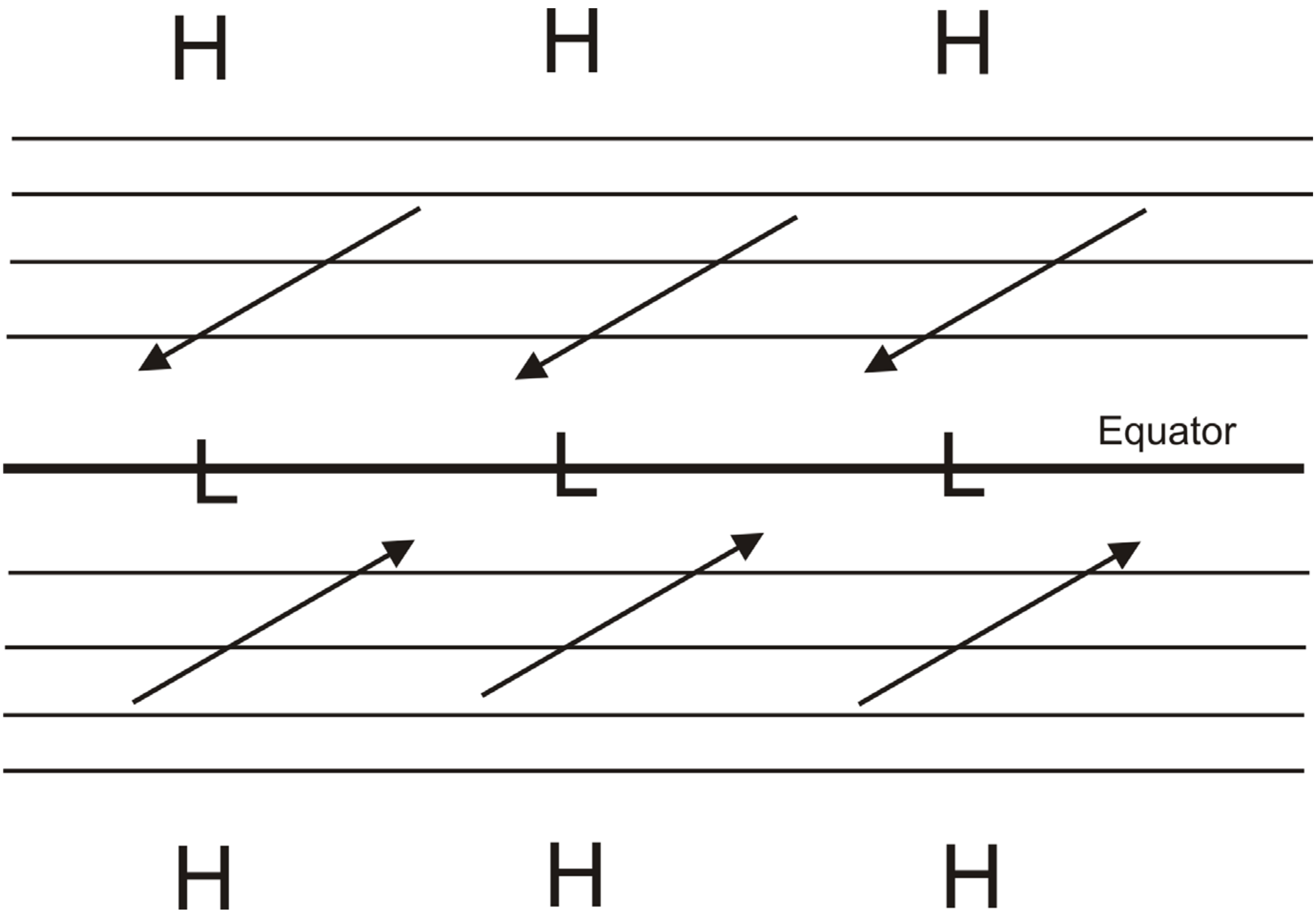
$$M_t = \Omega a^2$$

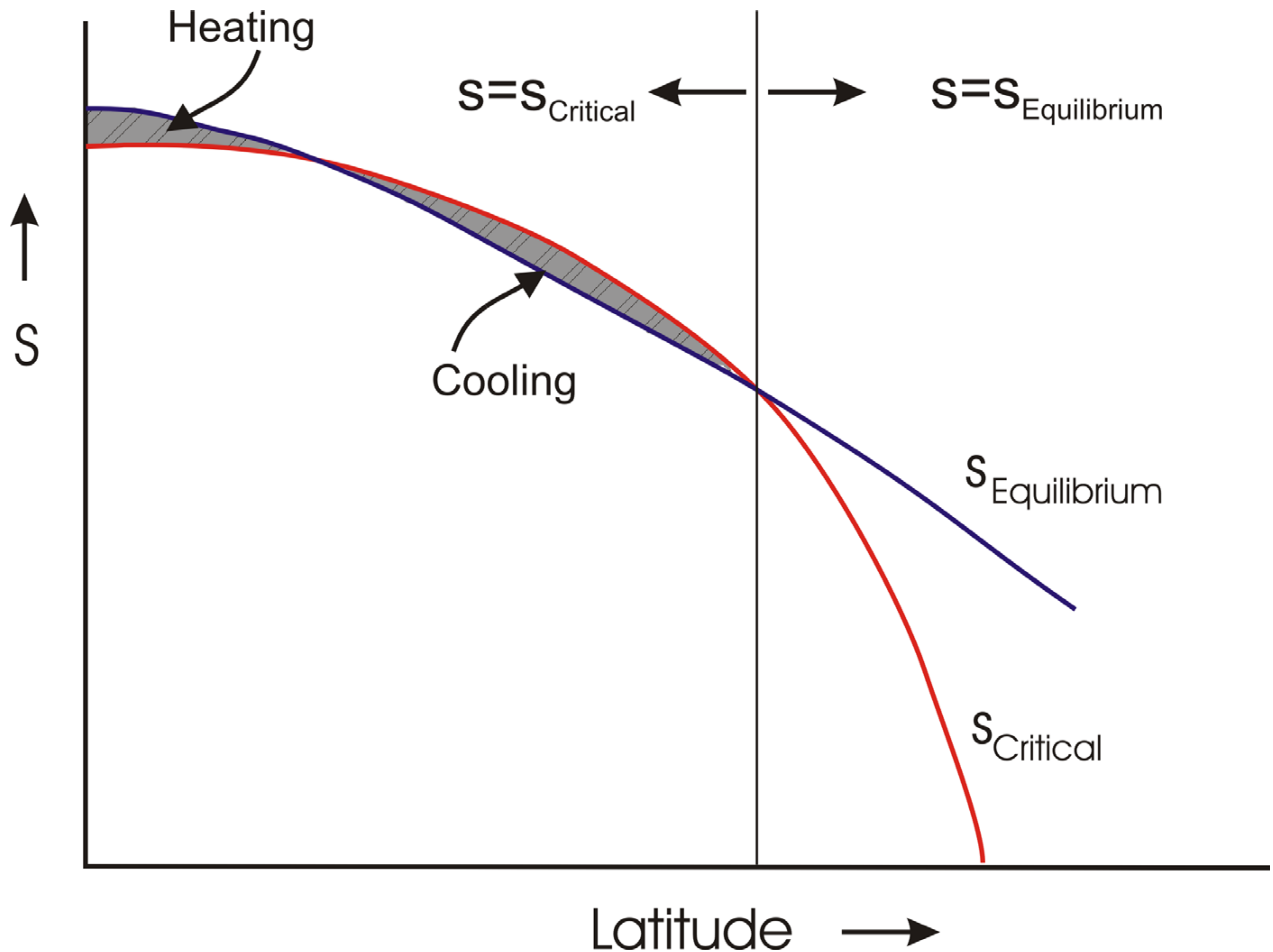
Integrate thermal wind equations ***down*** from tropopause:

$$M^2 = \Omega^2 a^4 + a^2 \cos^2\theta \cot\theta \left(T - T_t\right) \frac{\partial s^*}{\partial \theta}$$

$$\rightarrow \quad u \simeq \frac{\Omega a}{2}\left(\frac{1}{\cos^3\theta} - \cos\theta\right) + \frac{T - T_t}{2\Omega a \sin\theta} \frac{\partial s^*}{\partial \theta}$$

Note: u at surface always < 0 for supercritical entropy distribution


H
H
H
Equator
L
L
L
H
H
H



Heating
S=S Critical
S=S Equilibrium
S
Cooling
S Equilibrium
S Critical
Latitude

Critical entropy distribution:

$$\frac{T_s - T_t}{\Omega^2 a^2}\frac{\partial s^*}{\partial y} = \frac{1}{2}\left(\frac{1}{y^2} - 1\right)$$

$$\rightarrow \quad \frac{T_s - T_t}{\Omega^2 a^2} s^* = \frac{T_s - T_t}{\Omega^2 a^2} s_{eq}^* \left(1 - \frac{1}{2}\left(y + \frac{1}{y}\right)\right)$$

$$= \frac{T_s - T_t}{\Omega^2 a^2} s_{eq}^* \left(1 - \frac{1}{2}\left(\cos^2\theta + \frac{1}{\cos^2\theta}\right)\right)$$

Violation results in large-scale overturning circulation, known as the Hadley Circulation, that transports heat poleward and drives surface entropy gradient back toward its critical value

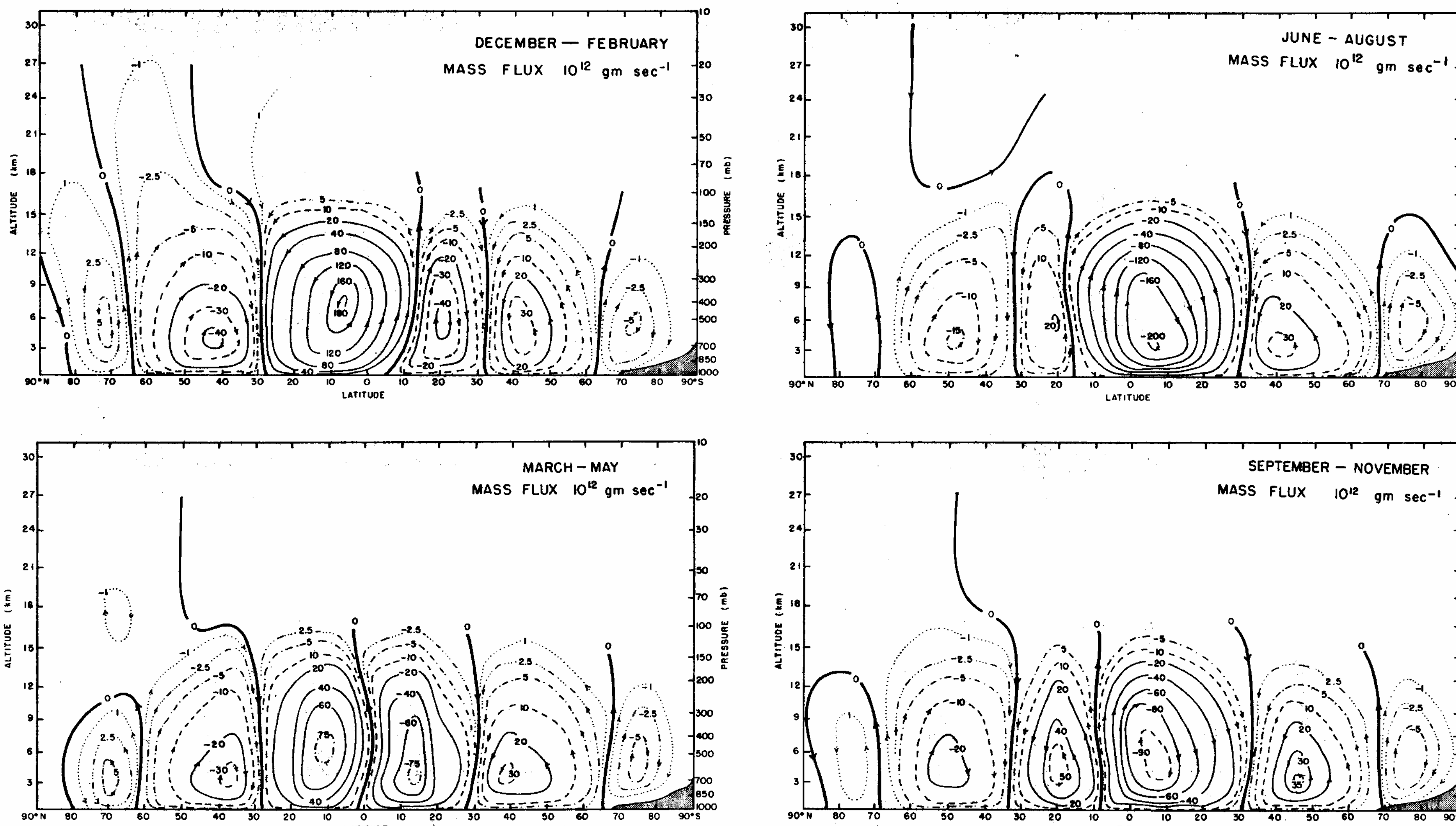


Figure 3.19

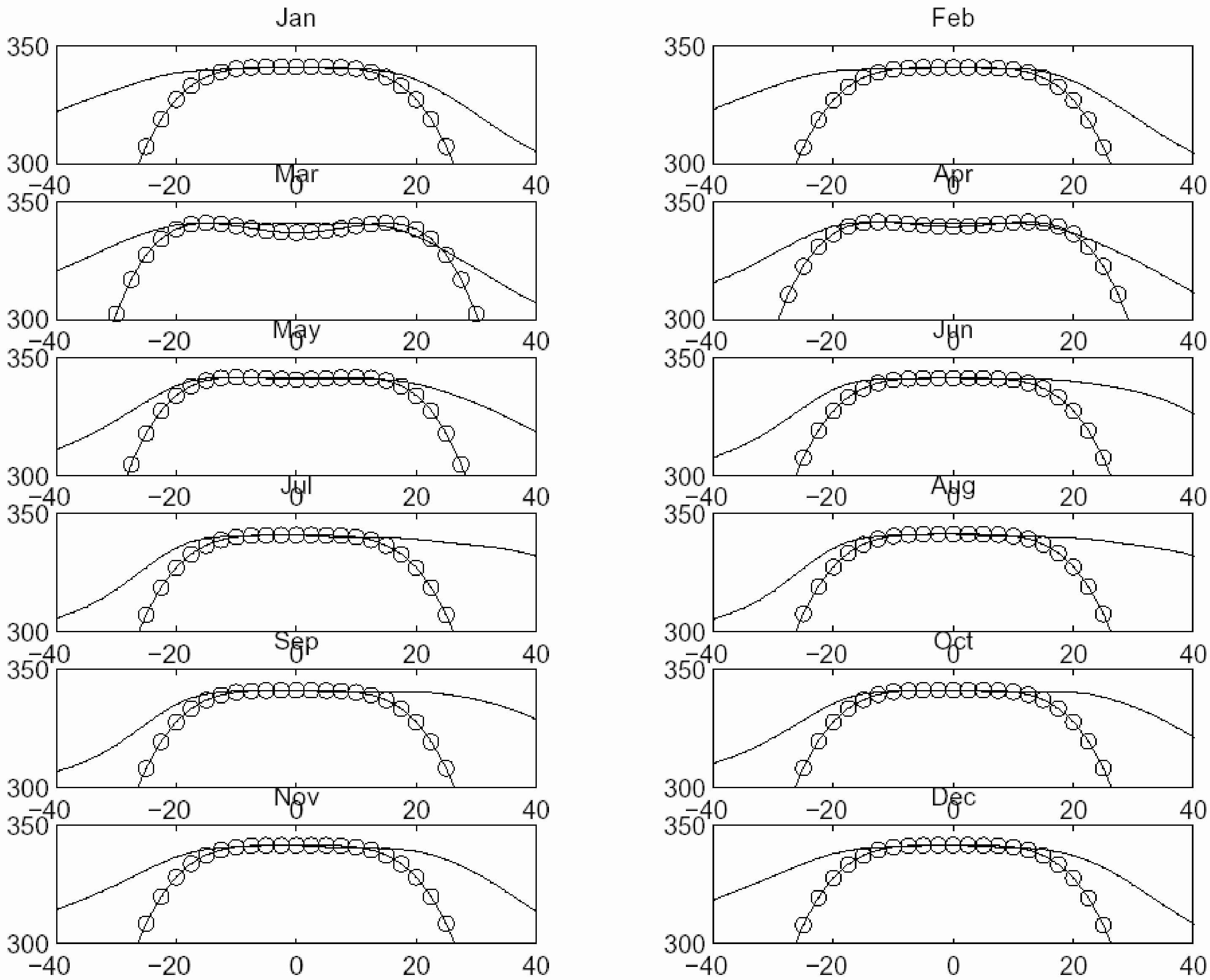
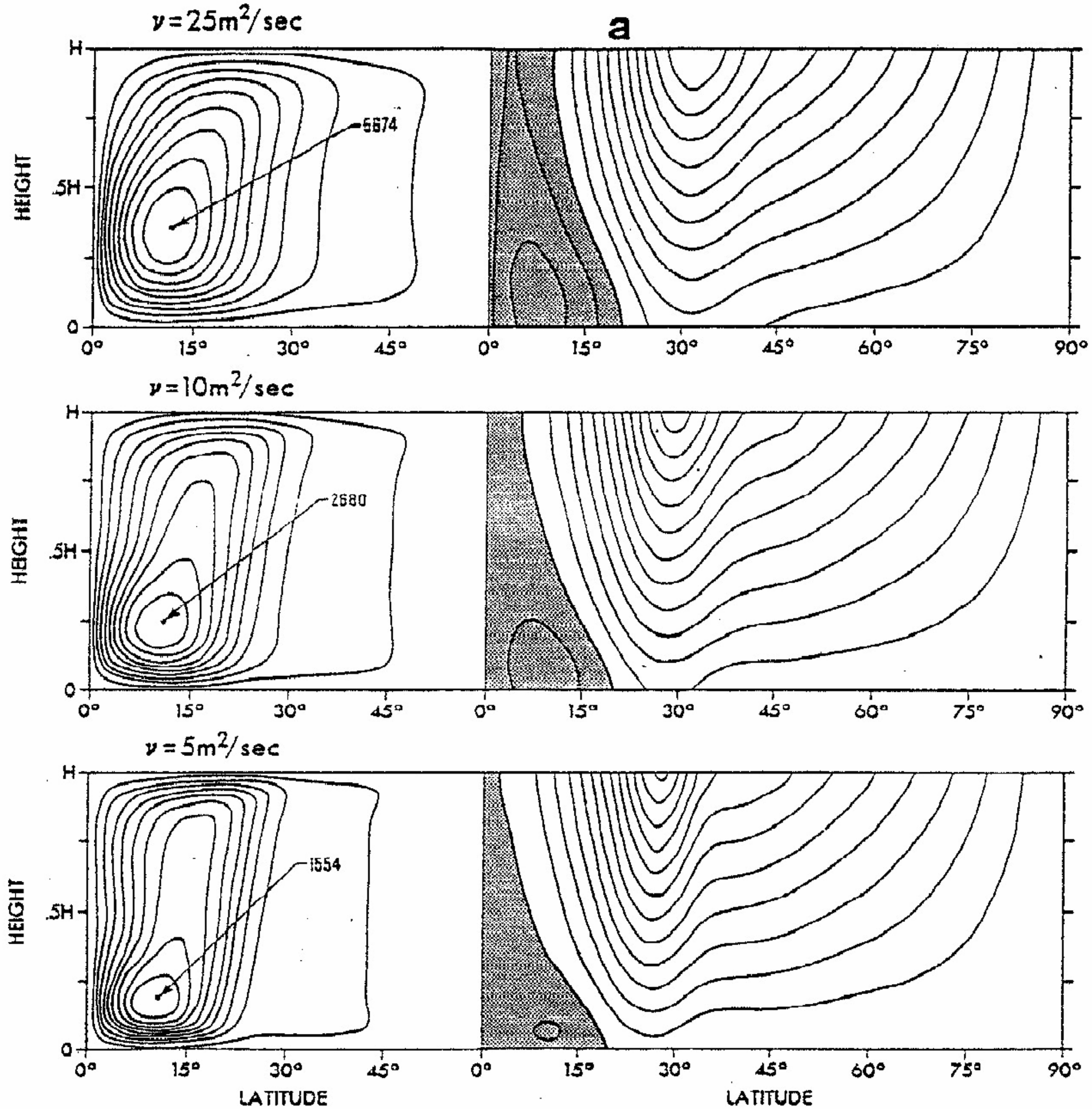
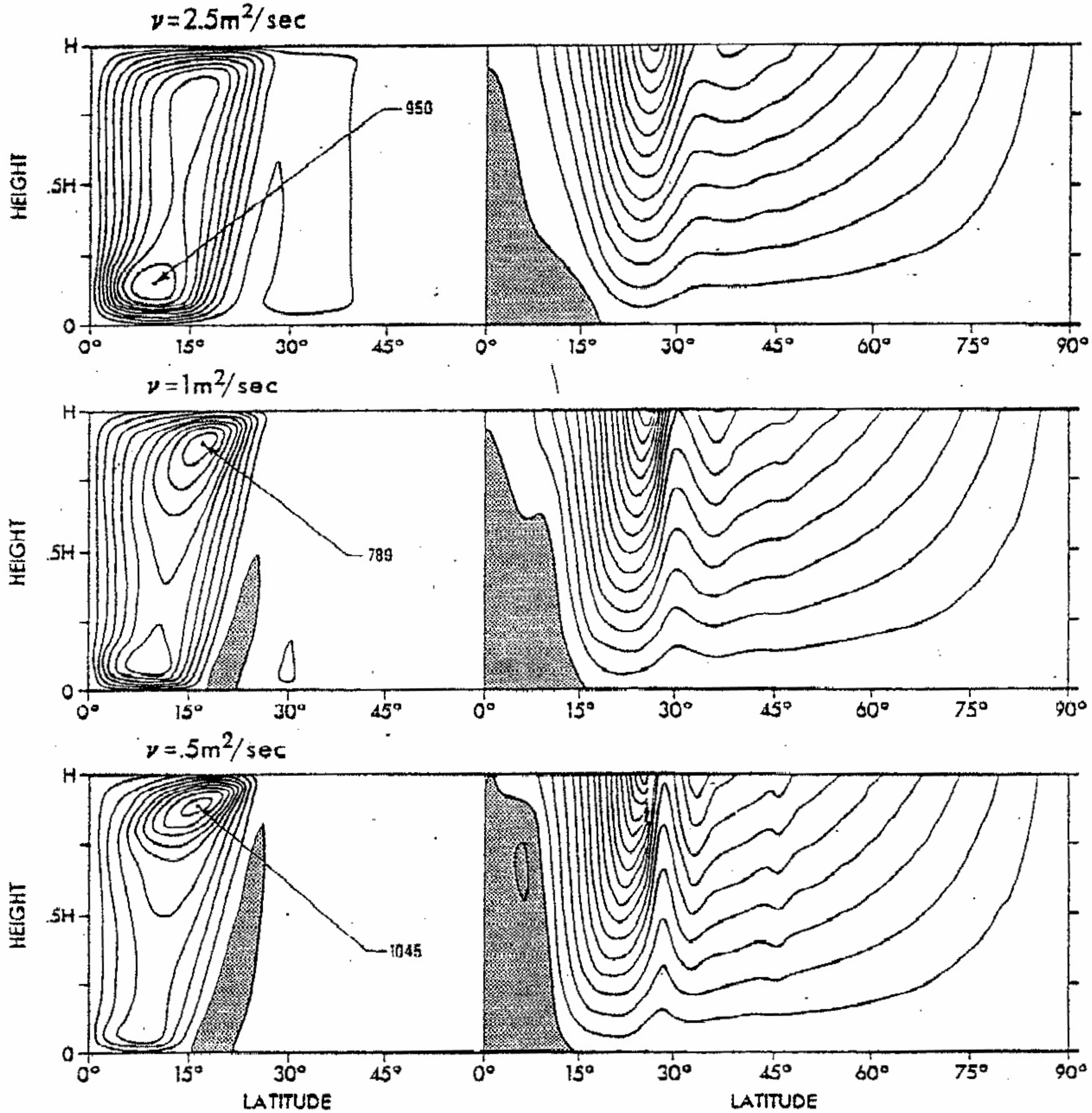


Figure 2: Critical (open circles) and observed (solid line) distributions of θ_{es} for every month at the 600 mb level as a function of latitude

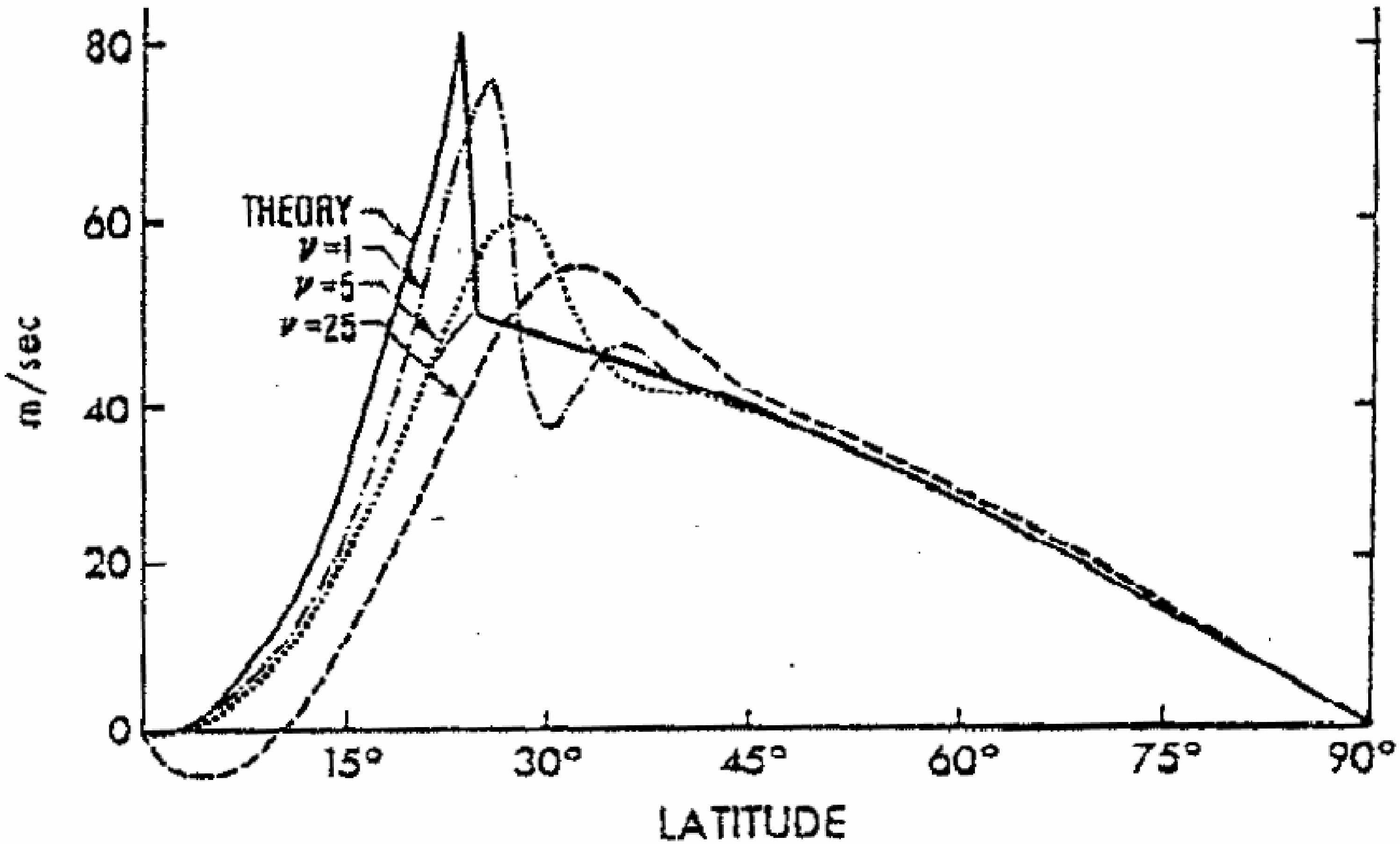
Simulations by Held and Hou (1980)



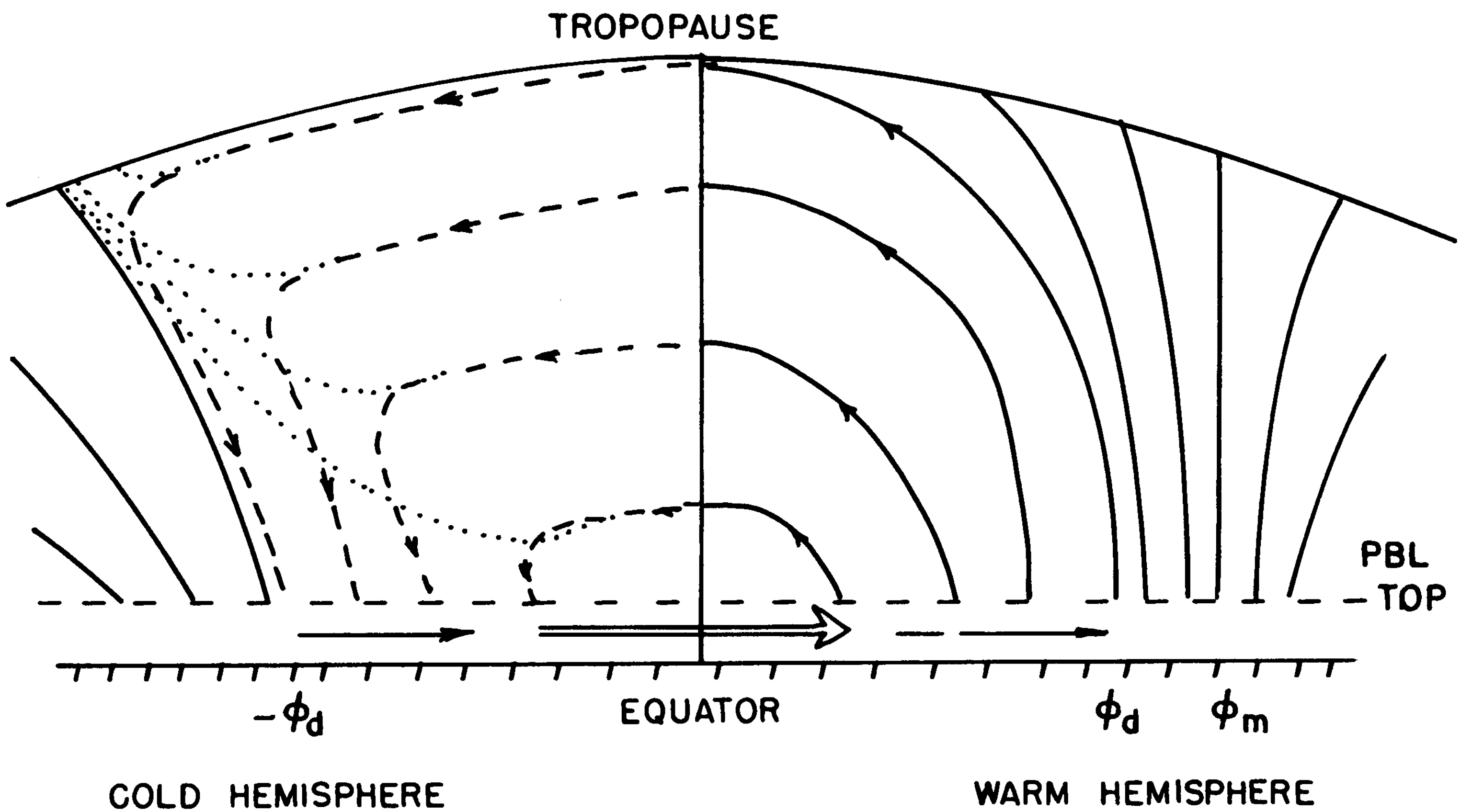


$\nu = 2.5 m^2/sec$
H
.5H
0
HEIGHT
950
0° 15° 30° 45°
0° 15° 30° 45° 60° 75° 90°
$\nu = 1 m^2/sec$
789
$\nu = .5 m^2/sec$
1045
LATITUDE

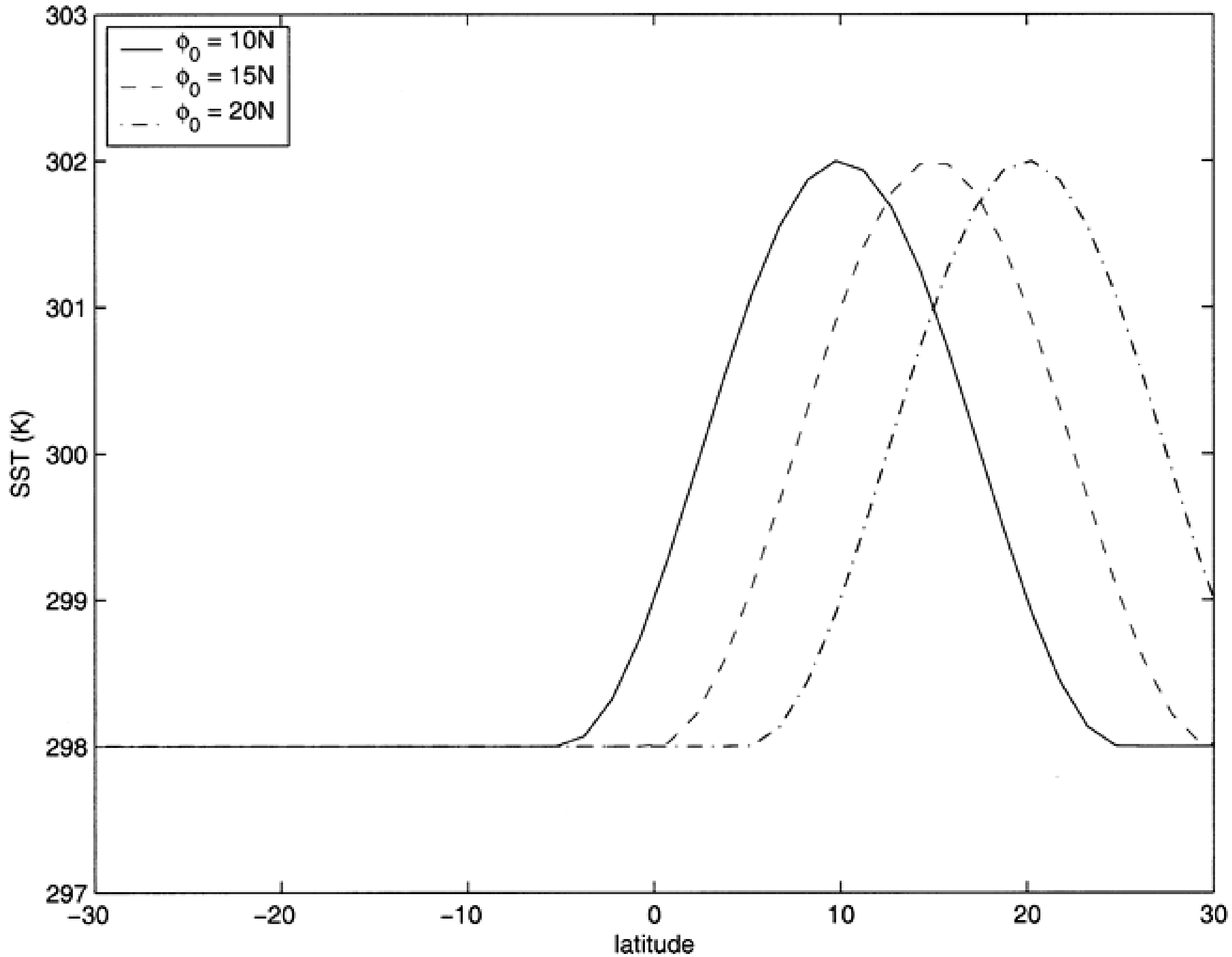
Zonal wind at tropopause



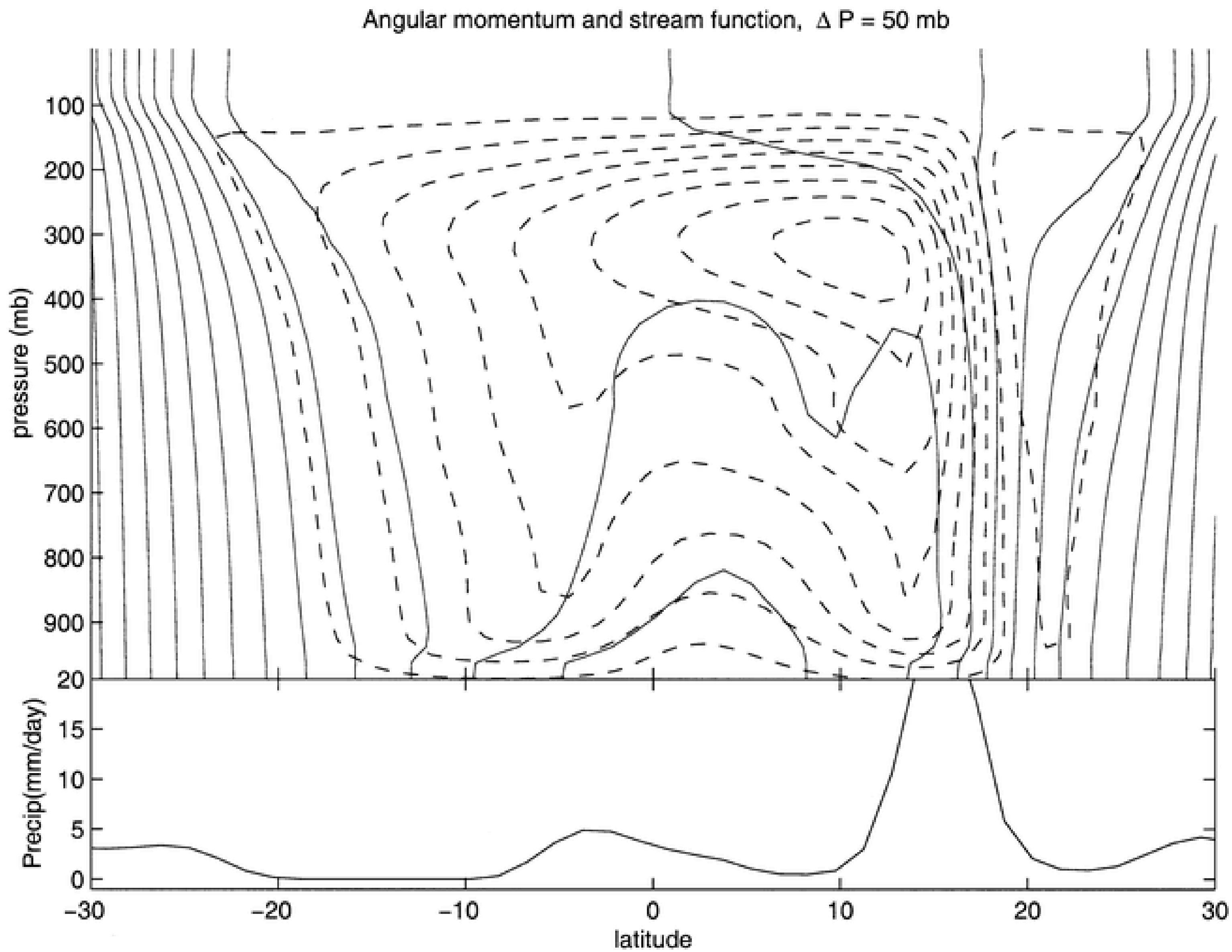
The cross-equatorial Hadley Circulation



Numerical simulation using a 2-D primitive equation model with parameterized convection (Pauluis, 2004)



Imposed sea surface temperature



Angular momentum and stream function, Δ P = 50 mb
pressure (mb)
100
200
300
400
500
600
700
800
900
20
15
10
5
0
Precip(mm/day)
−30
−20
−10
0
10
20
30
latitude

Why is there ascent on cold side of equator?

Meridional pressure gradient above the PBL:

$$\frac{\partial \varphi}{\partial y} = -\mathbf{V} \bullet \nabla v - 2\Omega \sin\theta\, u$$

$$\cong -\frac{1}{2}\frac{\partial v^2}{\partial y} - 2\Omega \sin\theta\, u$$

Hydrostatic:

$$\varphi_{PBL} = \varphi - R\frac{\Delta p_{pbl}}{p}\left(T_v\right)_{pbl}$$

$$\rightarrow \quad \frac{\partial \varphi_{pbl}}{\partial y} = \frac{\partial \varphi}{\partial y} - R\frac{\Delta p_{pbl}}{p}\left(\frac{\partial T_v}{\partial y}\right)_{pbl}$$

$$= -\frac{1}{2}\frac{\partial v^2}{\partial y} - 2\Omega\sin\theta\, u - R\frac{\Delta p_{pbl}}{p}\left(\frac{\partial T_v}{\partial y}\right)_{pbl}$$

PBL momentum:

$$V \bullet \nabla v - F_v = -\left(\frac{\partial \varphi}{\partial y}\right)_{pbl} - 2\Omega \sin\theta\, u_{pbl}$$

$$\rightarrow \frac{1}{2}\left(\frac{\partial v^2}{\partial y}\right)_{pbl} - F_v \cong \frac{1}{2}\left(\frac{\partial v^2}{\partial y}\right) + R\frac{\Delta p_{pbl}}{p}\left(\frac{\partial T_v}{\partial y}\right)_{pbl} + 2\Omega \sin\theta \left(u - u_{pbl}\right)$$

$$F_v \cong -C_D \frac{|\mathbf{V}|}{h} v$$

F_v dominant when $h <\sim C_D \Delta y$

$$\left(C_D \simeq 10^{-3}\right)$$

Thin, frictionally dominated PBL:

$$C_D \frac{|\mathbf{V}|}{h} v \cong R \frac{\Delta p_{pbl}}{p} \left(\frac{\partial T_v}{\partial y} \right)_{pbl} + 2\Omega \sin\theta \left(u \right)$$

u constrained by angular momentum conservation:

$$u_{min} = \frac{\Omega a}{\cos\theta} \left[\sin^2\theta - \sin^2\theta_0 \right],$$

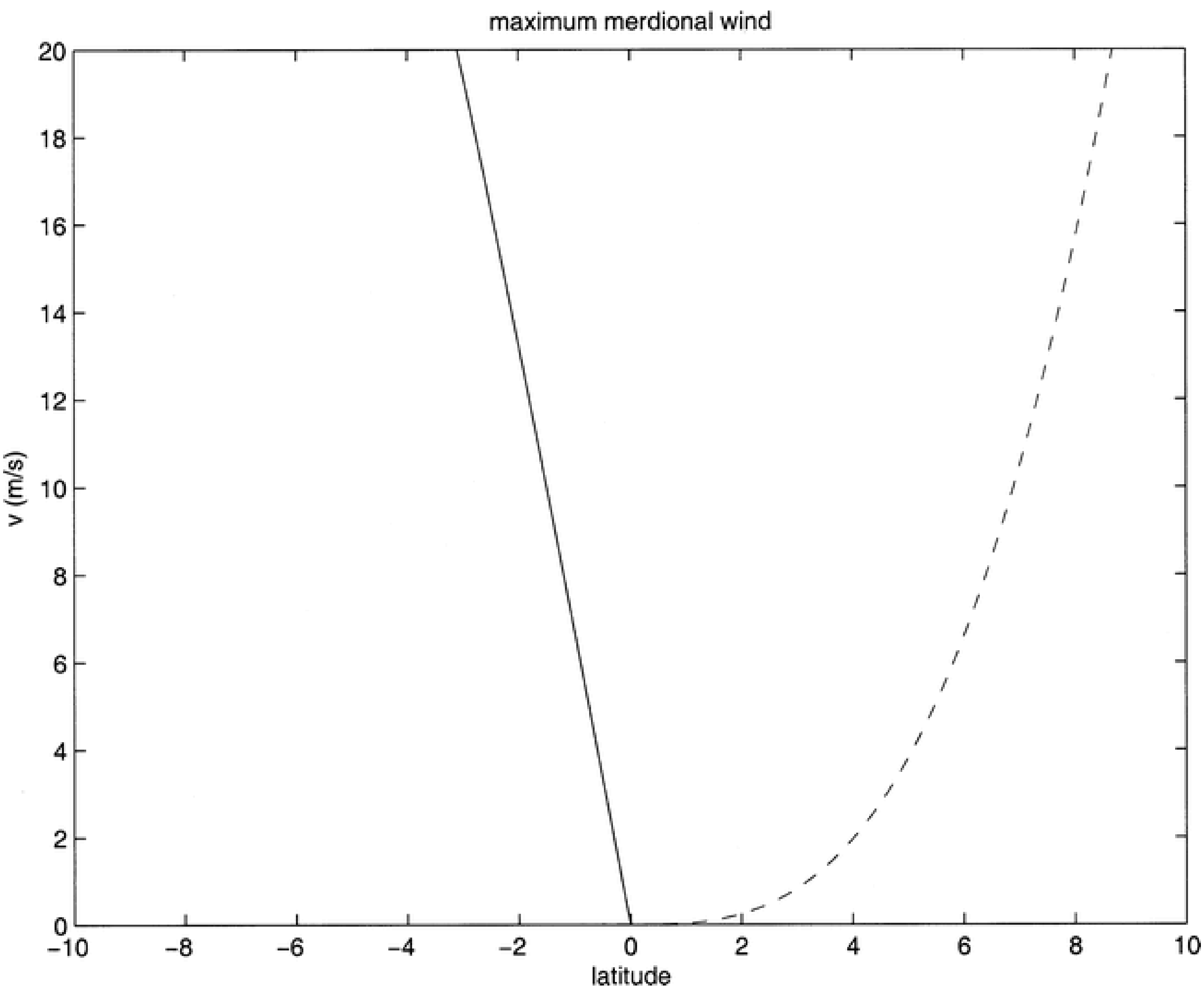
$$u_{max} = \frac{\Omega a}{\cos\theta} \sin^2\theta$$

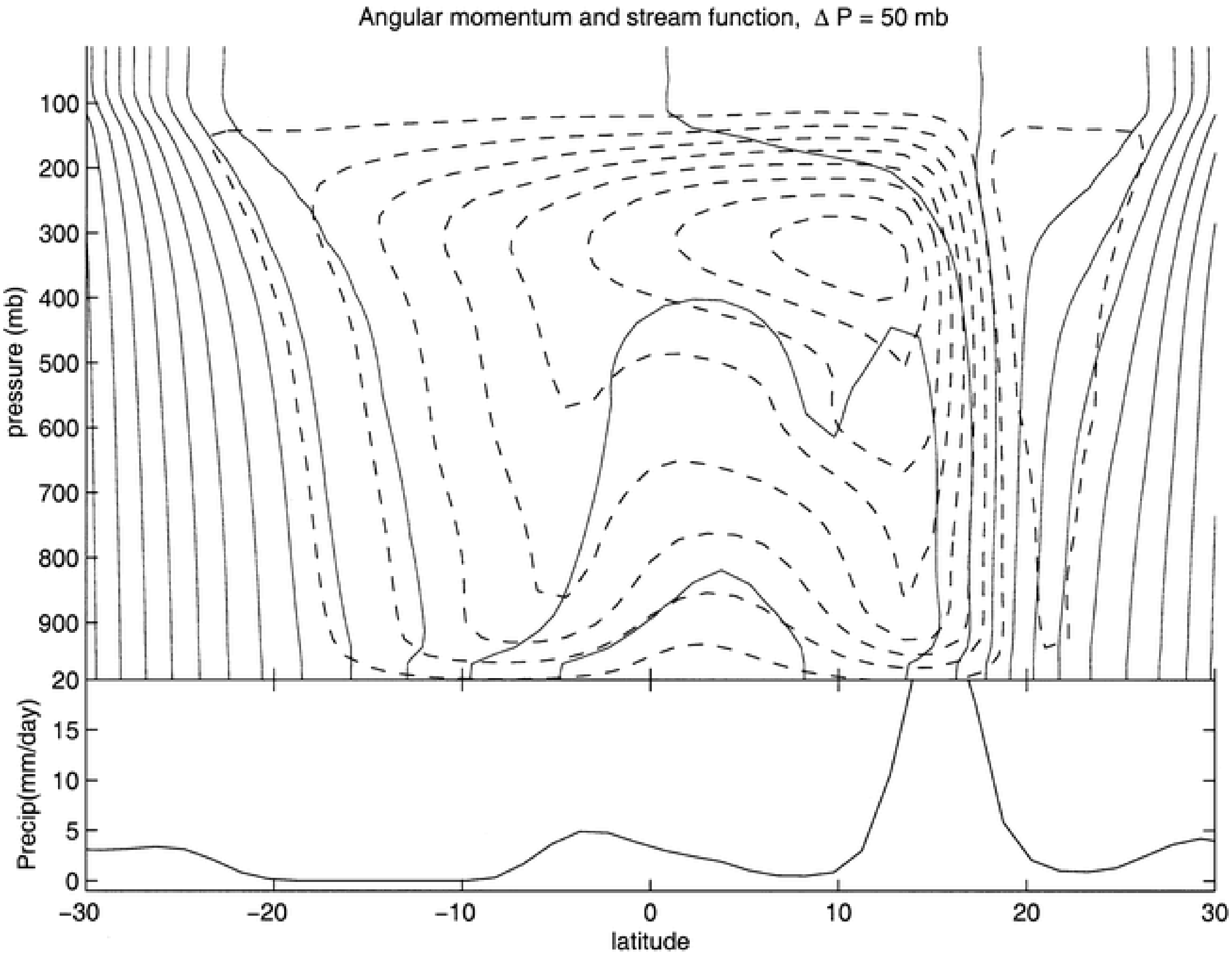
Cold side of equator:

$$C_D \frac{|\mathbf{V}|}{h} v \cong R \frac{\Delta p_{pbl}}{p} \left(\frac{\partial T_v}{\partial y} \right)_{pbl} + 2\Omega^2 a \tan\theta \left[\sin^2\theta - \sin^2\theta_0 \right]$$

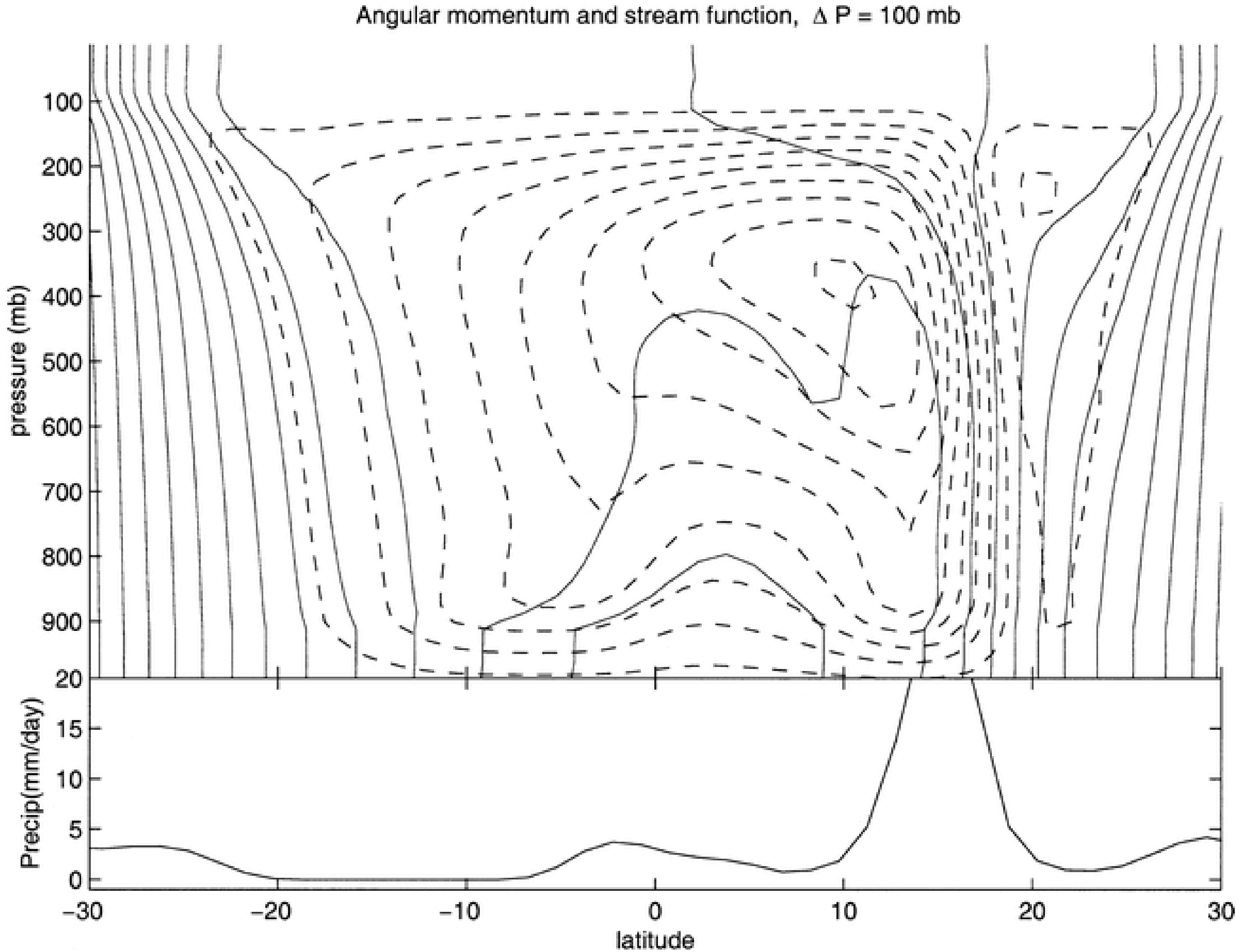
Warm side of equator:

$$C_D \frac{|\mathbf{V}|}{h} v \cong R \frac{\Delta p_{pbl}}{p} \left(\frac{\partial T_v}{\partial y} \right)_{pbl} + 2\Omega^2 a \tan\theta \sin^2\theta$$

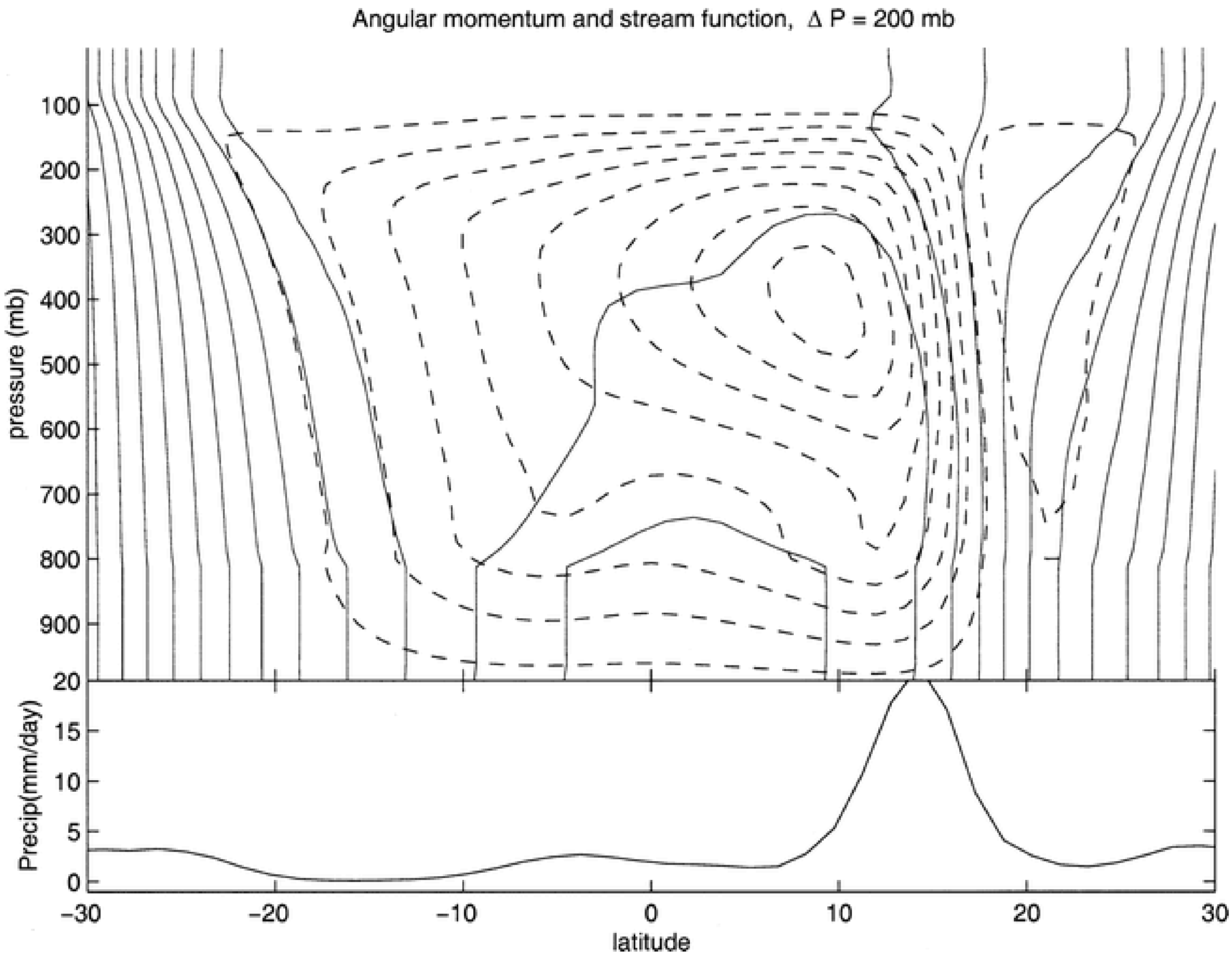

maximum merdional wind
v (m/s)
latitude
20
18
16
14
12
10
8
6
4
2
0
−10
−8
−6
−4
−2
0
2
4
6
8
10



$$\Delta p_{pbl} = 50\ mb$$

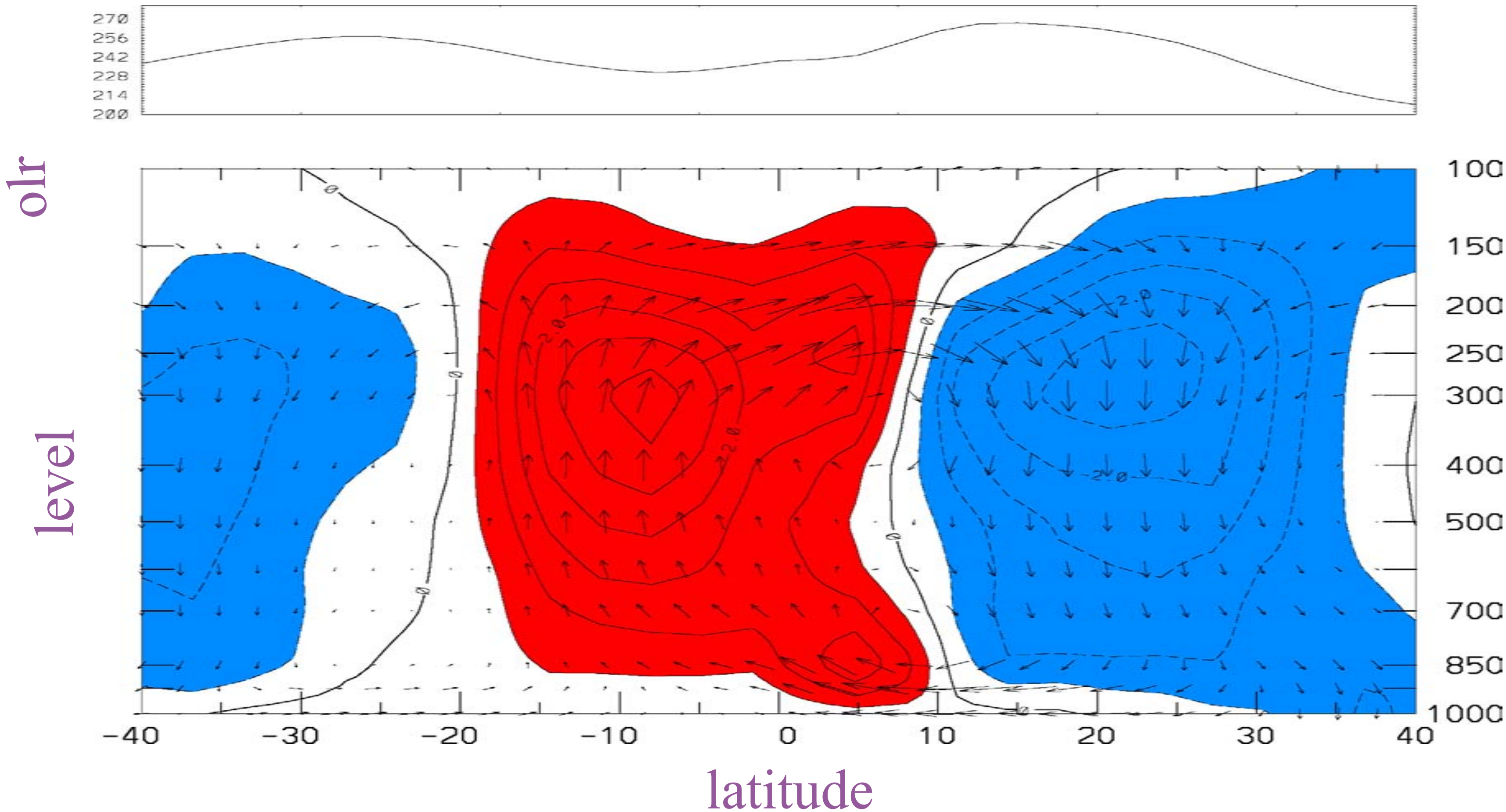


$$\Delta p_{pbl} = 100\ mb$$



$$\Delta p_{pbl} = 200\ mb$$

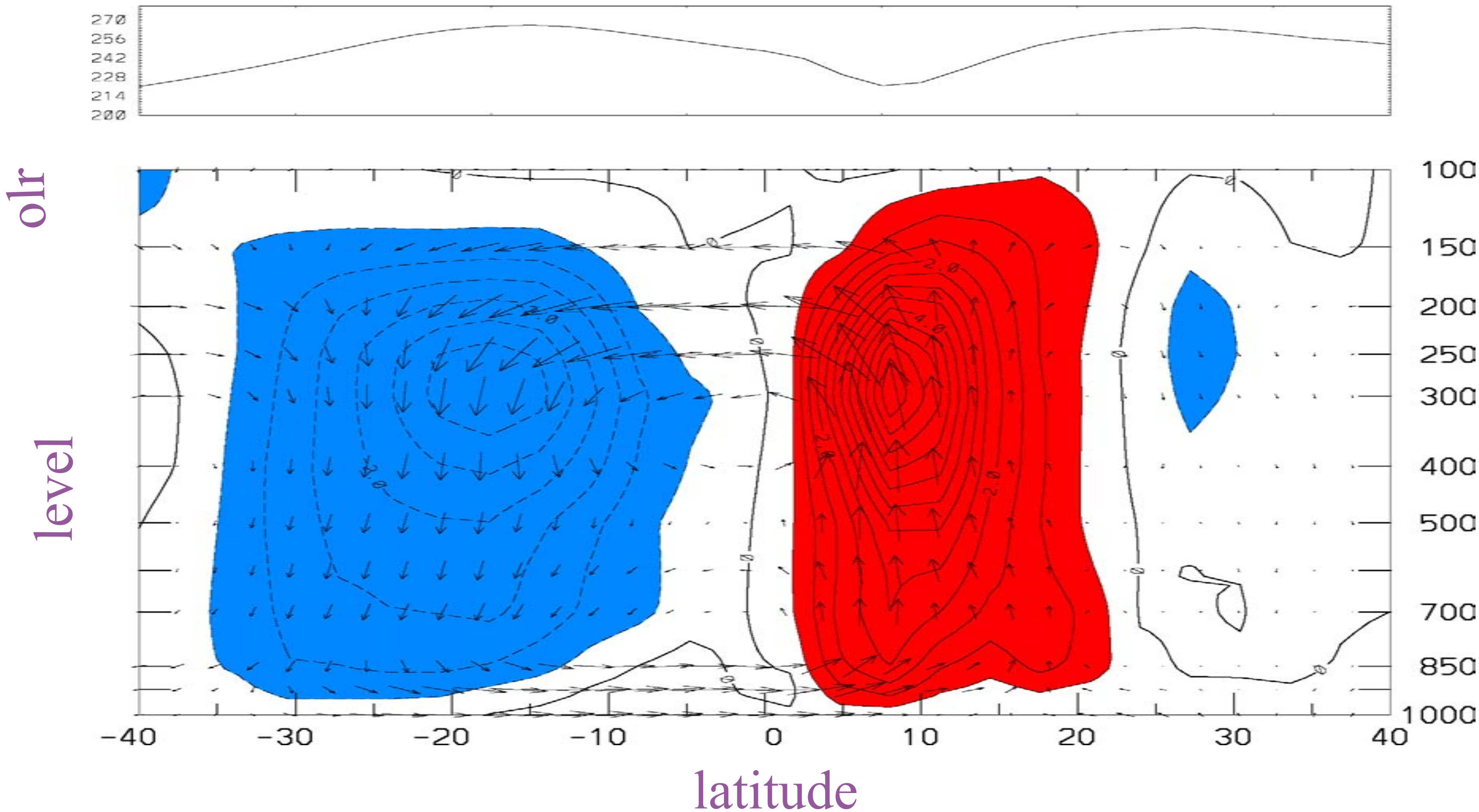
January Zonal Mean OLR, Vertical and Meridional Wind, 1979-1993 from ECMWF



Contour interval 1 mm s^{-1}

Shading Red Positive (Upward)

July Zonal Mean OLR, Vertical and Meridional Wind, 1979-1993 from ECMWF

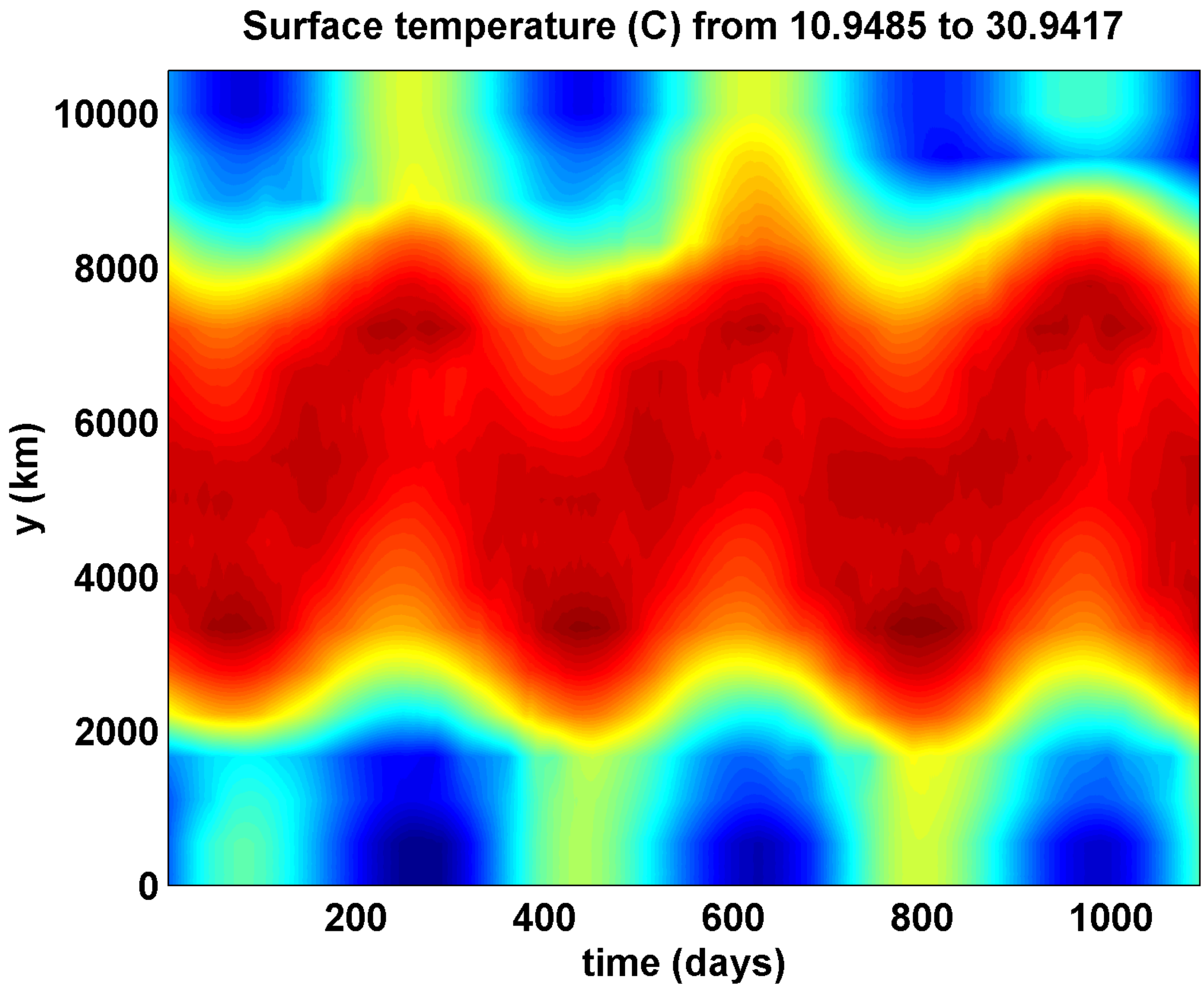


Contour interval 1 mm s^{-1}

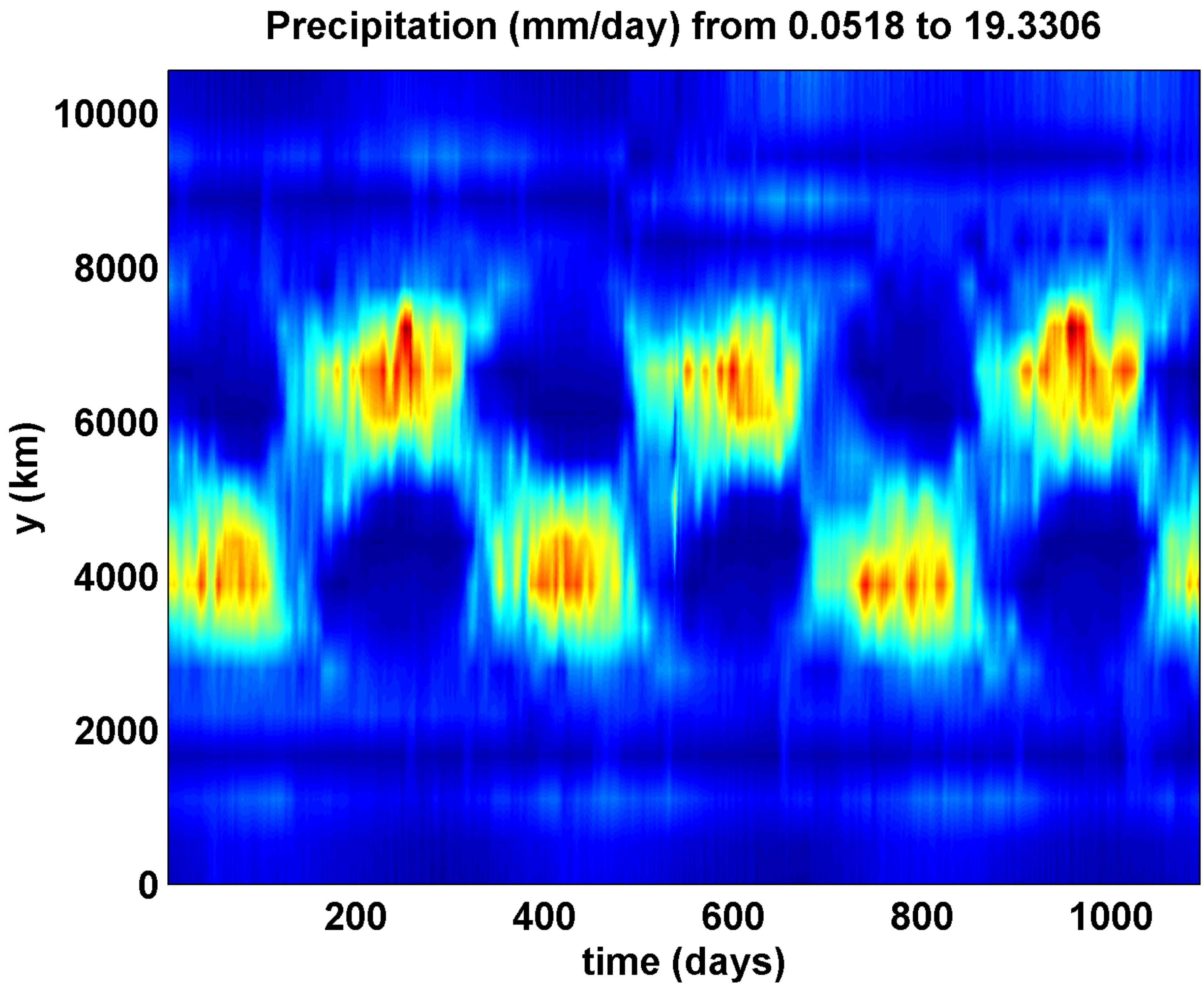
Shading Red Positive (Upward)

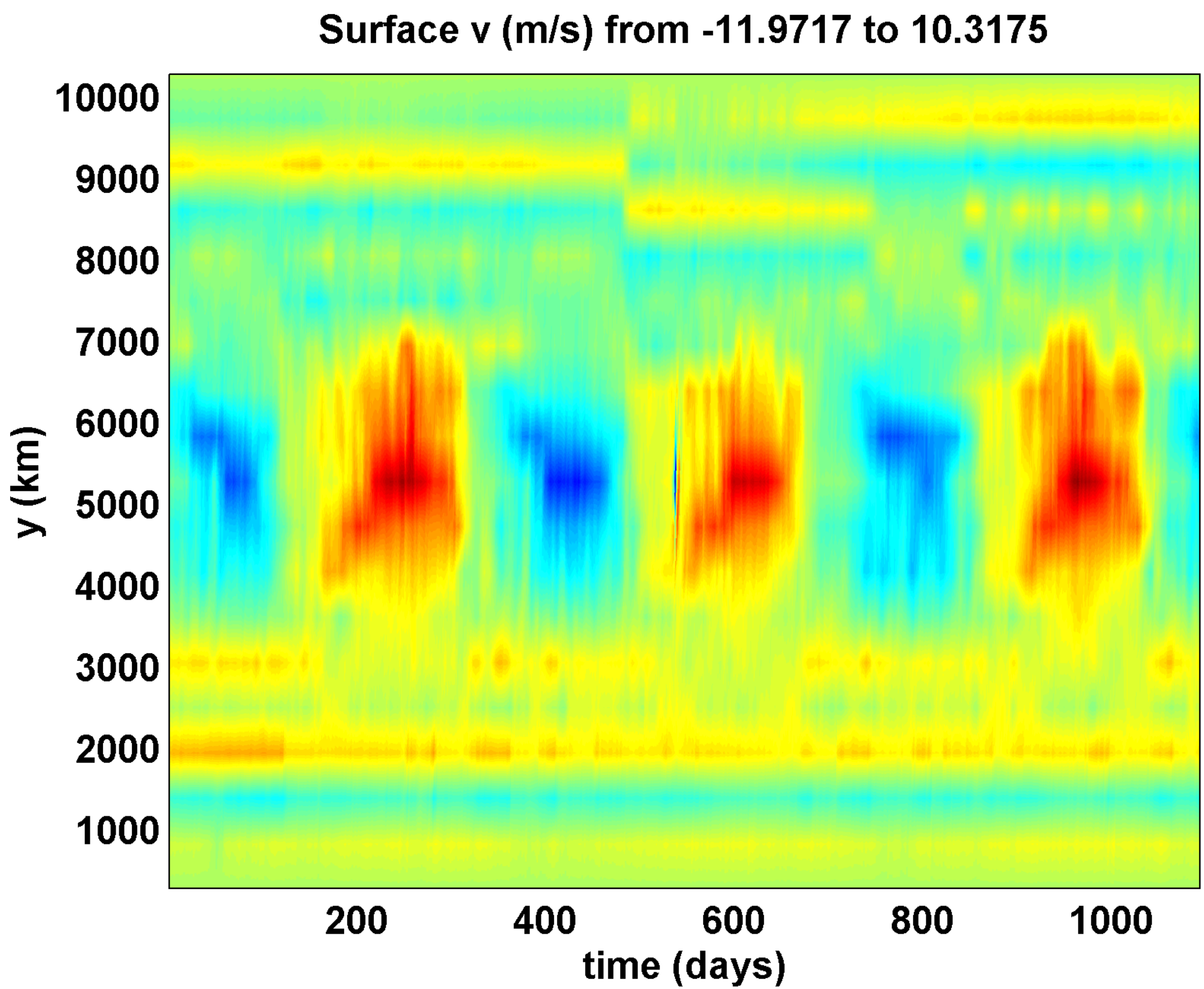
Two-D primitive equation model

- Parameterizations of
  - convection
  - fractional cloudiness
  - radiation
  - surface fluxes
- Ocean mixed layer energy budget
- Model forced by annual cycle of solar radiation
- Available for class projects

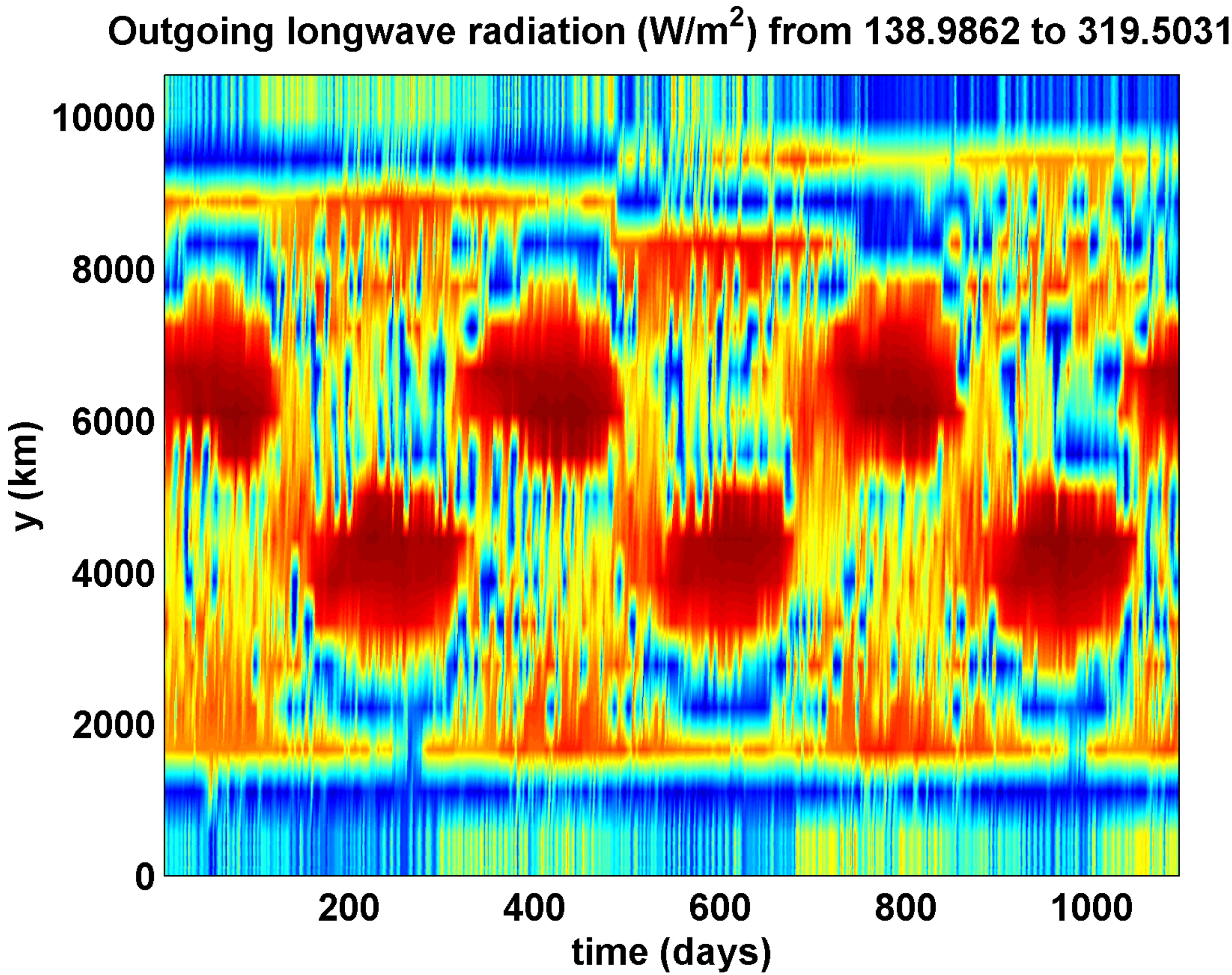


Surface temperature (C) from 10.9485 to 30.9417
y (km)
10000
8000
6000
4000
2000
0
200
400
600
800
1000
time (days)

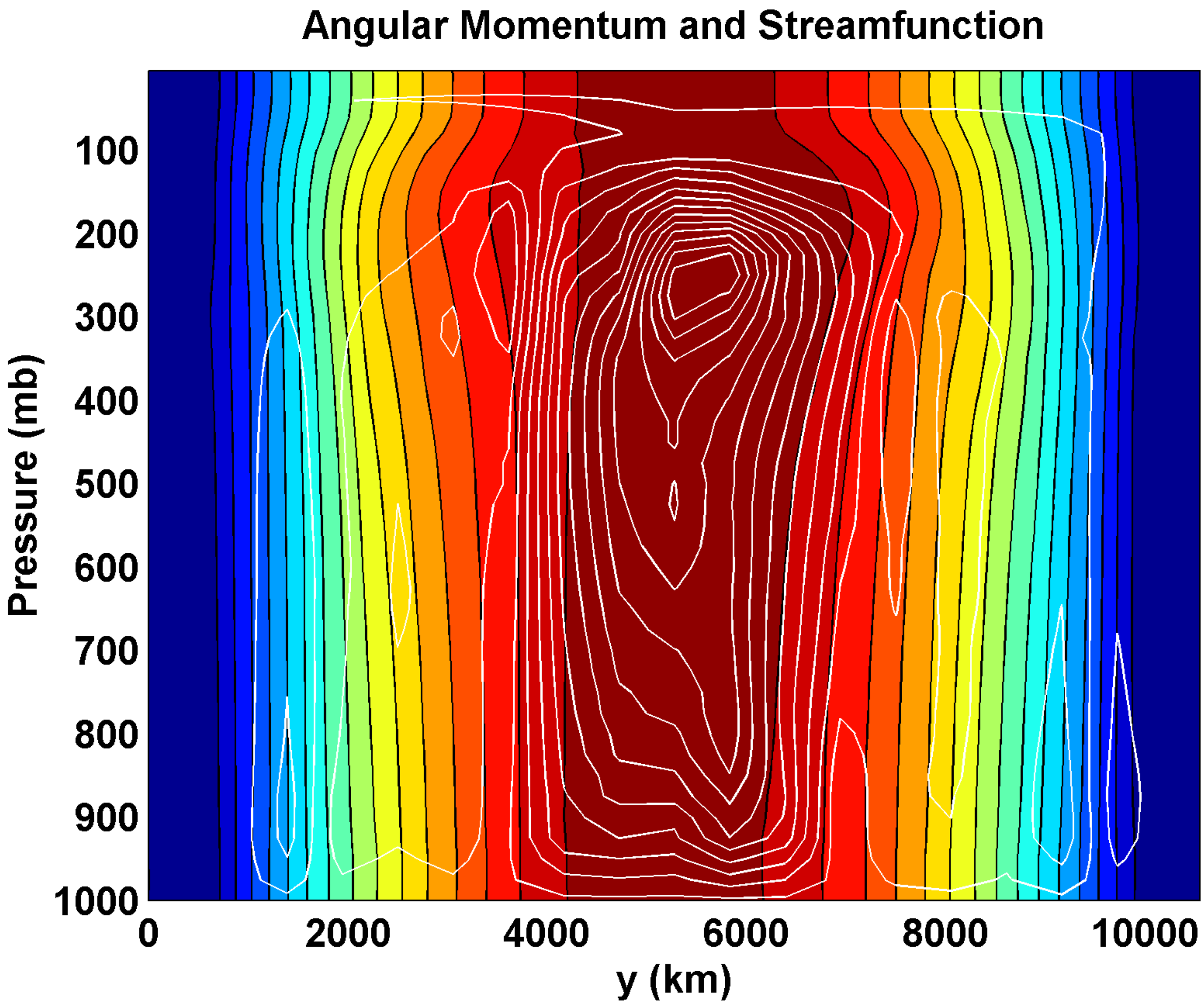

Precipitation (mm/day) from 0.0518 to 19.3306
y (km)
10000
8000
6000
4000
2000
0
200
400
600
800
1000
time (days)

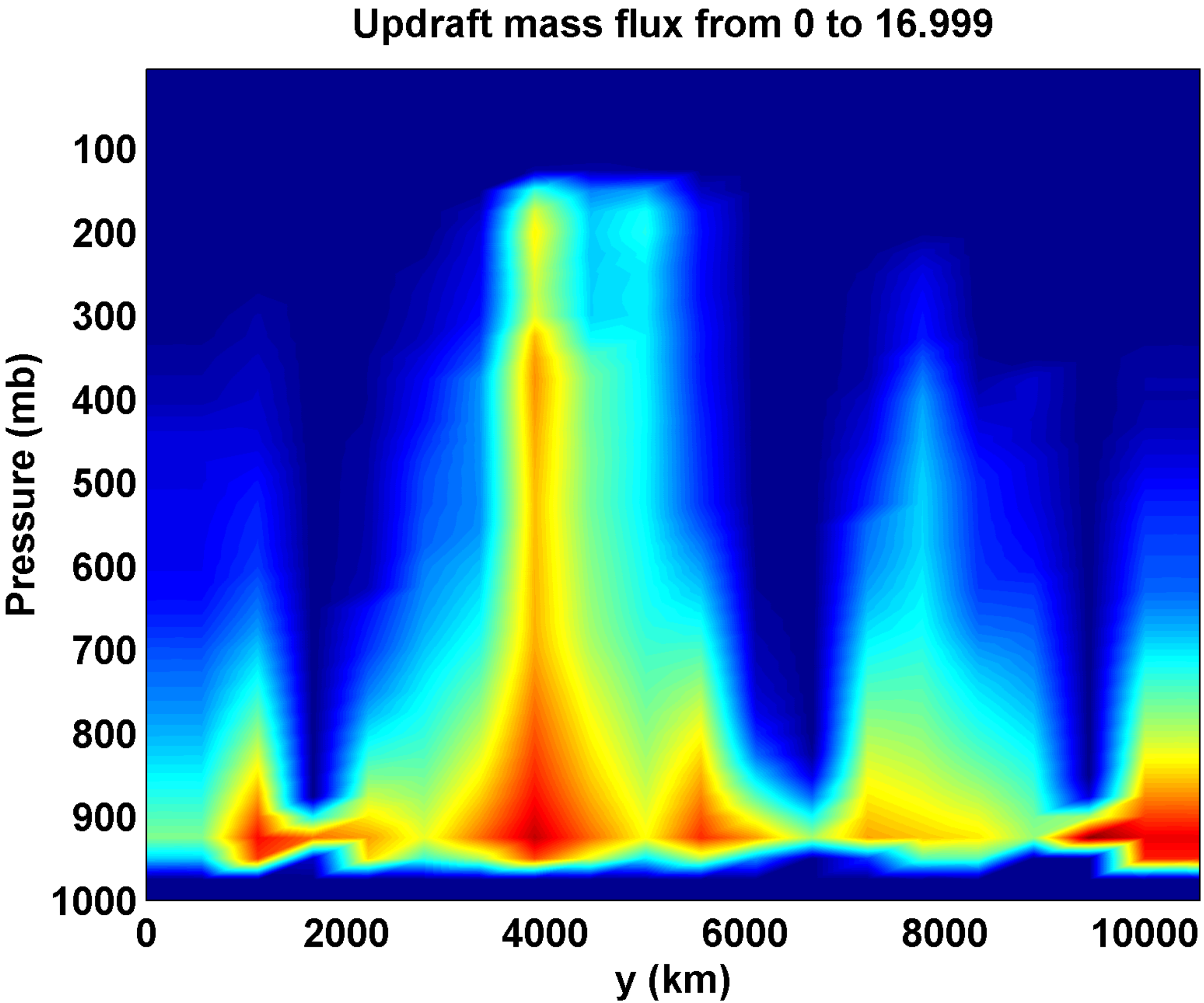


Surface v (m/s) from -11.9717 to 10.3175
y (km)
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
200
400
600
800
1000
time (days)

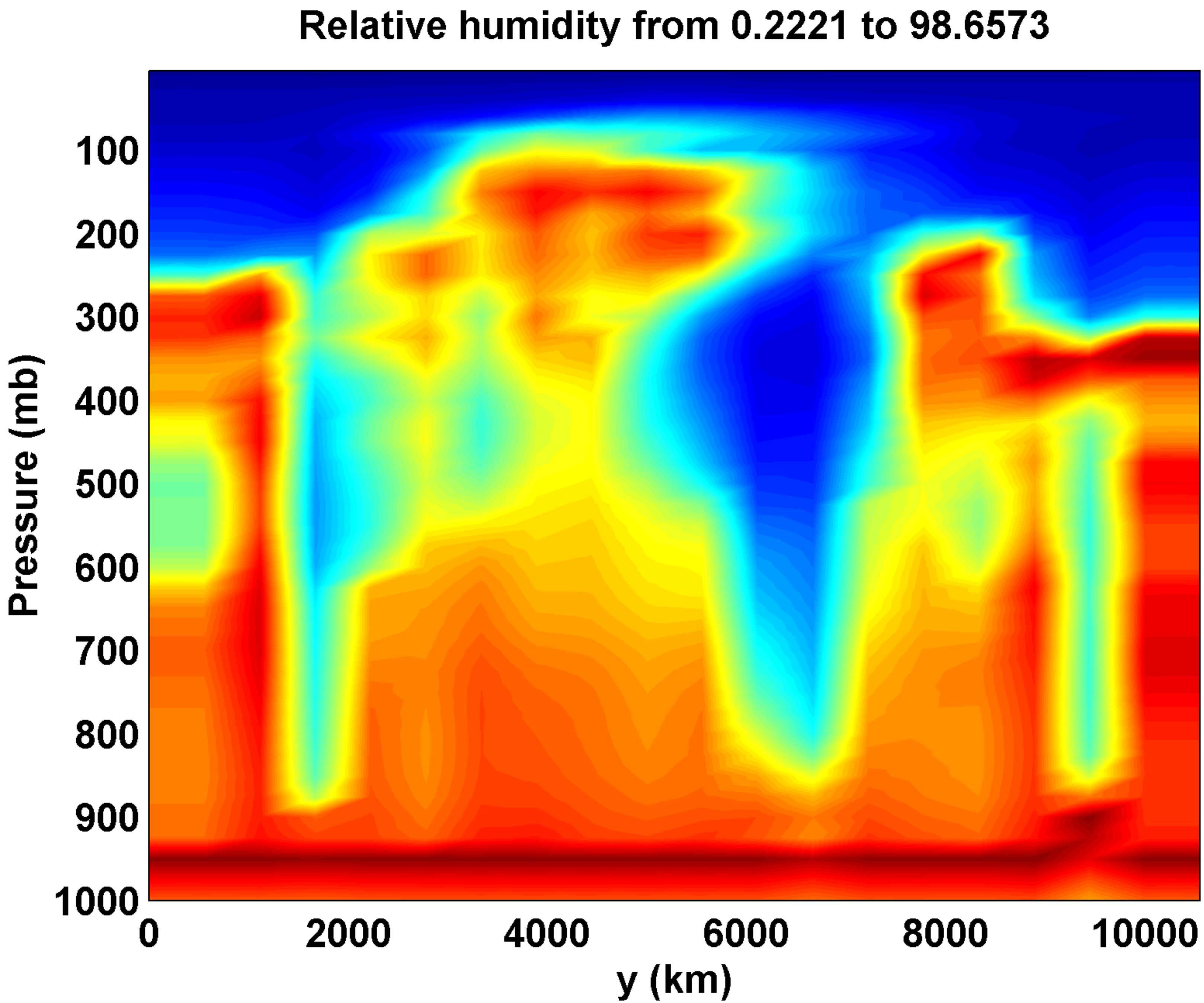


Outgoing longwave radiation (W/m^2) from 138.9862 to 319.5031
10000
8000
6000
4000
2000
0
y (km)
200
400
600
800
1000
time (days)


Angular Momentum and Streamfunction
Pressure (mb)
100
200
300
400
500
600
700
800
900
1000
0
2000
4000
6000
8000
10000
y (km)



Updraft mass flux from 0 to 16.999
Pressure (mb)
100
200
300
400
500
600
700
800
900
1000
0
2000
4000
6000
8000
10000
y (km)



Relative humidity from 0.2221 to 98.6573
Pressure (mb)
100
200
300
400
500
600
700
800
900
1000
0
2000
4000
6000
8000
10000
y (km)