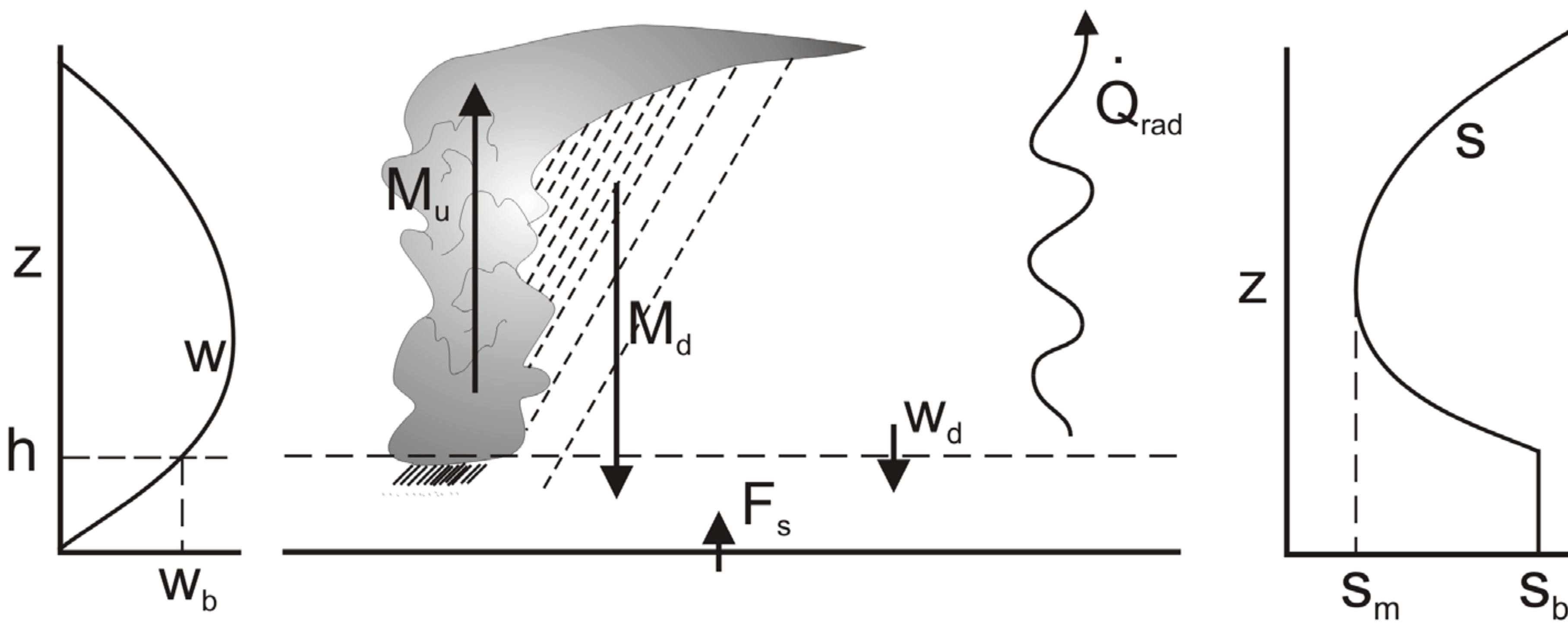
Relationship between moist convection and Hadley flow

Boundary layer entropy equilibrium hypothesis

(Raymond, 1995)



$$h\frac{\partial s_b}{\partial t} \approx 0 = F_s - \left(M_d + \left(1-\sigma\right) w_d\right)\left(s_b - s_m\right)$$

Mass:

$$M_u - M_d - \left(1-\sigma\right) w_d = w_b$$

$$\rightarrow M_u = w_b + \frac{F_s}{s_b - s_m} \qquad (1)$$

Free troposphere heat balance:

$$\left(M_u - M_d - w\right)\mathbf{S} = \dot{Q}_{cool},$$

$$\mathbf{S} \equiv c_p \frac{T}{\theta}\frac{\partial \theta}{\partial z}$$

Convective downdraft:

$$M_d = \left(1 - \varepsilon_p\right) M_u$$

$$\rightarrow \boxed{\varepsilon_p M_u = w + \dot{Q}_{cool} / \mathbf{S}} \qquad (2)$$

Combine (1) and (2) $\quad Let \quad w_b = \gamma w$

$$w = \frac{1}{1-\gamma\varepsilon_p}\left[\frac{\varepsilon_p F_s}{s_b - s_m} - \frac{\dot{Q}_{cool}}{\mathbf{S}}\right],$$

$$M_u = \frac{1}{1-\gamma\varepsilon_p}\left[\frac{F_s}{s_b - s_m} - \frac{\gamma\dot{Q}_{cool}}{\mathbf{S}}\right]$$

Note that $\quad M_u > w$

Radiative-convective equilibrium: w=0

$$\rightarrow F_s = \frac{\dot{Q}_{cool}\left(s_b - s_m\right)}{\mathbf{S}\varepsilon_p},$$

$$M_u = \frac{\dot{Q}_{cool}}{\mathbf{S}\varepsilon_p}.$$

Define

$$\left(F_s\right)_{eq} \equiv \frac{\dot{Q}_{cool}\left(s_b - s_m\right)}{\mathbf{S}\varepsilon_p}$$

Then

$$w=\frac{\varepsilon_p}{1-\gamma\varepsilon_p}\left[\frac{F_s}{s_b-s_m}-\frac{\left(F_s\right)_{eq}}{\left(s_b-s_m\right)_{eq}}\right]$$

Surface fluxes: $F_s \cong C_k\,|\mathbf{V}|\left(s_0^*-s_b\right)$

$w > 0$ if

- $F_s > F_{eq}$
- $(s_b - s_m) < (s_b - s_m)_{eq}$
- $Q_{cool} < (Q_{cool})_{eq}$

Note also that we must have $M_u \geq 0$ so

in circumstances under which (1) and (2) yield $M_u < 0$

we take $M_u = 0$,

$$w_b = -F_s \big/ (s_b - s_m) \quad or \quad s_b - s_m = -F_s \big/ w_b$$

$$w = -\dot{Q}_{cool} \big/ \mathbf{S}$$

radiative-subsidence balance

Weak Temperature Gradient Approximation (WTG)
Sobel and Bretherton, 2000

- Ignore time dependence of T above PBL
- Determine w from aforementioned equations
- Determine vorticity from w
- Determine T by inverting balanced flow