DISSIPATIVE HEATING AND HURRICANE INTENSITY

Marja Bister and Kerry A. Emanuel*

Program in Atmospheres, Oceans, and Climate
Massachusetts Institute of Technology
Cambridge, MA 02139

(Submitted to the *Meteorology and Atmospheric Physics)*

15 June 1997

* *Corresponding author address:* Room 54–1620, Massachusetts Institute of Technology, Cambridge, MA 02139

ABSTRACT

Dissipative heating has not been accounted for in either numerical simulations of hurricanes or in theories for the maximum intensity of hurricanes. We argue that the bulk of dissipative heating occurs in the atmospheric boundary layer near the radius of maximum winds and, using both theory and numerical simulation, show that dissipative heating increases maximum wind speeds in tropical cyclones by about 20%.

1. Introduction

Numerical prediction of tropical cyclone tracks has improved enormously over the past few decades, but there is still little skill in forecasting storm intensity change (Elsberry et al., 1992). Part of the problem of forecasting hurricane intensity may rest with the inability of current forecast models to resolve the storms' inner core, leading to failure to predict the maximum wind speeds. Yet much of the failure to forecast hurricane intensity change may be blamed on our relative lack of understanding of the basic physical processes that control intensity.

It is by now well established that the basic thermodynamic cycle of the hurricane limits the maximum intensity that can be achieved. Work on delimiting energetic and thermodynamic bounds on intensity was begun by Miller (1958) and continued by Emanuel (1986, 1988, 1995a, 1997), and Holland (1997). The theoretical maximum intensity provides an excellent prediction of the actual maximum intensity of storms simulated in axisymmetric numerical models (Rotunno and Emanuel, 1987; Emanuel, 1995b). Yet the intensities of most real storms fall below the theoretical bound (e.g., see Schade, 1994), owing to three-dimensional interactions with the surrounding atmosphere, to local sea surface cooling induced by the storms themselves and, perhaps, to uncertainty in the theoretical bound arising from lack of knowledge of the sea surface exchange coefficients of heat, moisture, and momentum at high wind speeds. Even so, DeMaria and Kaplan (1994) found that an approximate measure of potential intensity, based on sea surface temperature alone, could be used as a statistically significant predictor of tropical cyclone intensity change.

The purpose of this short note is to point out that all of the extant theory and numerical simulations have neglected the thermodynamic energy source arising from dissipative heating, and to demonstrate that when included, the upper theoretical intensity bound and the actual intensity achieved in numerical simulations increase appreciably.

2. Dissipative heating

Frictional dissipation of kinetic energy ultimately occurs at molecular scales. The frictional terms in the momentum equations have the form

$$\frac{\partial}{\partial x_j}\left(\nu \frac{\partial u_i}{\partial x_j}\right), \tag{1}$$

where u_i is the ith component of the velocity and ν is the kinematic viscosity. In the kinetic energy equation, (1) is multiplied by u_i. The result may be expressed, after a little manipulation, by

$$\frac{\partial}{\partial x_j}\left(\nu \frac{\partial}{\partial x_j}\left(\frac{1}{2}u_i^2\right)\right) - \nu\left(\frac{\partial u_i}{\partial x_j}\right)^2. \tag{2}$$

Summing over the 3 components of velocity, the first term in (2) represents the diffusion of kinetic energy, and may have either sign, while the second is the dissipation of kinetic energy. Since total energy must be conserved, the dissipative heating term in the thermodynamic equation will be

$$\nu\left(\frac{\partial u_i}{\partial x_j}\right)^2. \tag{3}$$

Many numerical models represent the effect of turbulence on momentum by a term of the form of (1) except that ν represents an eddy viscosity, which may depend on both the velocities and their gradients. Thus ν is replaced by a term of the form ν_{ij}. The rest of the derivation of the dissipative heating term is the same, resulting in a term in the temperature equation of the form

$$\nu_{ij}\left(\frac{\partial u_i}{\partial x_j}\right)^2, \tag{4}$$

with summation over both indices. While (4) applies in the interior of the fluid flow, the surface stress is often represented by a bulk aerodynamic formula of the form

$$\nu\frac{\partial u_i}{\partial x_3}|_0 = C_D u_i\sqrt{u_1^2+u_2^2}, \tag{5}$$

where C_D is a drag coefficient and x_3 is the direction normal to the surface. (This applies only to u_1 and u_2.) Then, at the lowest model level, the frictional term in the thermodynamic equation is

$$\frac{C_D}{h}\left(u_1^2+u_2^2\right)^{3/2}, \tag{6}$$

where h is the altitude above sea level of the lowest model grid point.

3. Effect of frictional dissipation on upper-bound calculations

a. Dynamical derivation

Here we follow the derivations presented by Emanuel (1986, 1995), adding dissipative heating.

For an axisymmetric vortex in gradient and hydrostatic balance, and for which the temperature lapse rate is moist adiabatic along angular momentum surfaces, the thermal wind equation may be written (Emanuel, 1986)

$$\frac{1}{r_b^2} = \frac{1}{r_t^2} - 2c_p(T_s - T_o)\frac{1}{f^2R^3}\frac{\partial \ln \theta_e^*}{\partial R}, \tag{7}$$

where r_b and r_t are the (physical) radii of angular momentum surfaces at the top of the boundary layer and at the tropopause, respectively, c_p is the heat capacity at constant pressure, T_s and T_o are the surface temperature and temperature at the tropopause, respectively, θ_e^* is the saturation equivalent potential temperature, f is the Coriolis parameter, and R is the potential radius, defined so that

$$fR^2 = fr^2 + 2rV, \tag{8}$$

where V is the azimuthal velocity.

In a mature hurricane, the anticyclone is well developed at the tropopause and so angular momentum surfaces flair out to very large radius. Thus we may use the approximation $r_t \to \infty$ in (7):

$$\frac{1}{r_b^2} \simeq -2c_p(T_s - T_o)\frac{1}{f^2R^3}\frac{\partial \ln \theta_e^*}{\partial R}. \tag{9}$$

To find an expression for the maximum azimuthal wind velocity, we first find a separate expression for the gradient of $\ln \theta_e^*$ with respect to angular momentum (or potential radius, R). In the boundary layer under the eyewall, we assume that, in the steady state, radial advections of angular momentum and equivalent potential temperature are balanced by their sink and source at the sea surface:

$$-\frac{f}{2}\frac{\partial \psi}{\partial p}\frac{\partial R^2}{\partial r} = r\frac{\partial \tau_v}{\partial p}, \tag{10}$$

and

$$-c_p\frac{\partial \psi}{\partial p}\frac{\partial \ln \theta_e}{\partial r} = \frac{1}{T_s}\left[\frac{\partial \tau_k}{\partial p} + \frac{\mathcal{D}}{g}\right], \tag{11}$$

where ψ is the mass streamfunction in the r-p plane, τ_v is the vertical turbulent flux of azimuthal momentum, τ_k is the turbulent enthalpy flux, and $\mathcal{D}$ is the dissipative heating rate, which was neglected in previous derivations. In (10), $\frac{f}{2}R^2$ is the angular momentum per unit mass. Taking $\psi = 0$ at the sea surface and assuming that R^2 (or, equivalently, V) and θ_e are constant with altitude in the boundary layer, (10) and (11) may be integrated through the depth of the boundary layer to give

$$\frac{f}{2}\psi_b\frac{\partial R^2}{\partial r} = r\tau_{v_0}, \tag{12}$$

$$c_p\psi_b\frac{\partial \ln \theta_e}{\partial r} = \frac{1}{T_s}\left[\tau_{k_0} + \frac{1}{g}\int_{p_b}^{p_0}\mathcal{D}dp\right], \tag{13}$$

where ψ_b is the mass streamfunction at the top of the boundary layer and p_0 and p_b are the surface pressure and pressure at the top of the boundary layer, respectively, and the surface fluxes are given by the aerodynamic flux formulae

$$\tau_{v_0} = -\rho C_D |\mathbf{V}| V, \tag{14}$$

$$\tau_{k_0} = \rho C_k |\mathbf{V}| (k^* - k). \tag{15}$$

Here ρ is the air density near the surface, C_D and C_k are the exchange coefficients for momentum and enthalpy, $|\mathbf{V}|$ is the magnitude of the surface wind speed, and k^* and k are the saturation enthalpy of the sea surface and the actual enthalpy of the boundary layer air, respectively.

Now we assume that the frictional heating of the boundary layer is given by (6), so that

$$\frac{1}{g} \int_{p_b}^{p_0} \mathcal{D} dp = \rho C_D |\mathbf{V}|^3. \tag{16}$$

Now divide (13) by (12) and use (14)–(16) to yield

$$-\frac{1}{R} \frac{\partial \ln \theta_e}{\partial R} = \frac{f}{c_p T_s} \frac{1}{rV} \left[\frac{C_k}{C_D} (k^* - k) + |\mathbf{V}|^2 \right]. \tag{17}$$

We next assume that in the eyewall region, θ_e in the boundary layer is equal to θ_e^* along angular momentum surfaces above the boundary layer, the condition of slantwise convective neutrality. Then (17) can be substituted into (9) to give

$$\frac{R^2}{r_b^2} \simeq 2 \frac{T_s - T_o}{T_s} \frac{1}{frV} \left[\frac{C_k}{C_D} (k^* - k) + |\mathbf{V}| \right]. \tag{18}$$

In the eyewall region we assume that $|\mathbf{V}| \gg fr$, so that (8) becomes, approximately,

$$fR^2 \simeq 2rV. \tag{19}$$

Eliminating r between (18) and (19) gives

$$V^2 \simeq \frac{T_s - T_o}{T_s} \left[\frac{C_k}{C_D}(k^* - k) + |\mathbf{V}|^2 \right]. \tag{20}$$

Finally, if we assume that $|\mathbf{V}| \simeq V$, then (20) becomes

$$\boxed{V^2 = \frac{T_s - T_o}{T_o} \frac{C_k}{C_D}(k^* - k).} \tag{21}$$

This is identical to expressions derived previously by Emanuel (1986, 1995) except that T_o instead of T_s appears in the denominator. Thus all the results of Emanuel (1986, 1995) follow but with the coefficient C_k/C_D replaced by

$$\frac{T_s}{T_o} \frac{C_k}{C_D}. \tag{22}$$

The effect of including dissipative heating is identical to that of increasing the ratio of surface exchange coefficients, C_k/C_D, by the factor T_s/T_o. In hurricane environments, $T_s/T_o \approx 3/2$, so including dissipative heating is equivalent to increasing the enthalpy transfer coefficient by 50%.

b. Derivation from conservation principles

Consider the steady-state entropy balance in a control volume bounded by two closely spaced surfaces of constant entropy, angular momentum, and streamfunction, as shown in Figure 1. Import of entropy into the control volume through the top of the boundary layer must, in equilibrium, equal the export of entropy through an arbitrary surface of constant temperature, since, by definition, there is no lateral import or export of entropy:

$$(s\delta\psi)_0 = (s\delta\psi)_T,$$

where s is the specific entropy ($=c_p ln\theta_e$), ψ is the mass streamfunction, and the subscripts 0 and T denote evaluation at the top if the boundary layer, and at some absolute temperature, T, respectively. Since s and ψ surfaces coincide and are thus the same at the top and bottom of the control volume, it follows that

$$(\psi\delta s)_0 = (\psi\delta s)_T. \tag{23}$$

In the boundary layer, entropy advection must balance the sum of the reversible and irreversible entropy sources:

$$\rho\mathbf{V}\cdot\nabla s = \rho(\dot{s}_{rev} + \dot{s}_{irr}), \tag{24}$$

where $\dot{s}_{rev}$ and $\dot{s}_{irr}$ are the reversible and irreversible entropy sources, respectively, and ρ is the density. Integrating (24) over the control volume in Figure 1 gives

$$-2\pi(\psi\delta s)_0 = 2\pi\int_r^{r+\delta r}\int_0^z \rho\left(\dot{s}_{rev} + \dot{s}_{irr}\right)dzrdr. \tag{25}$$

We assume that above the boundary layer near the radius of maximum winds, the flow is entirely steady and adiabatic, and that the only reversible and irreversible entropy sources are in the boundary layer, so that the integrands in (25) vanish outside the boundary layer. (Outside the radius of maximum winds, where there is overall descent, radiative cooling is important in the steady state and the entropy advection outside the boundary layer will not be negligble.) We further assume that frictional dissipation accounts for the entire irreversible entropy source, and that the reversible entropy source is the heat flux from the ocean. Then, if the boundary layer is thin, (25) can be written

$$-2\pi(\psi\delta s)_0 = -2\pi(\psi\delta s)_T = \pi\delta r_0^2\rho_s\frac{C_k|\mathbf{V_s}|(k_0^* - k)}{T_s} + \pi\delta r_0^2\rho_s\frac{C_D|\mathbf{V_s}|^3}{T_s}, \qquad (26)$$

where we have made use of (23) and have used (15) for the surface heat flux and (16) for the dissipative heating. (Dividing these by temperature gives the respective entropy sources.) The subscripts s and 0 denote evaluation at the surface and at the top of the boundary layer, respectively.

From the first law of thermodynamics,

$$T\delta s = c_p\delta T + L_v\delta q - \alpha\delta p, \qquad (27)$$

where c_p is the heat capacity at constant pressure, L_v is the latent heat of vaporization, q is the specific humidity and α is the specific volume. Since the flow is steady, we may make use of the steady state momentum equations to write

$$\begin{aligned}\alpha\delta p &= \alpha\frac{\partial p}{\partial z}\delta z + \alpha\frac{\partial p}{\partial r}\delta r \\ &= \delta z[-g - \mathbf{V}\cdot\nabla w] + \delta r\left[\left(\frac{v^2}{r} + fv\right) - \mathbf{V}\cdot\nabla u\right].\end{aligned} \tag{28}$$

The advective terms can be re-written

$$(\mathbf{V}\cdot\nabla w)\delta z + (\mathbf{V}\cdot\nabla u)\delta r = \delta\left(\frac{1}{2}(u^2 + w^2)\right) + \frac{1}{\rho r}\zeta\delta\psi, \tag{29}$$

where

$$\zeta = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial r},$$

and

$$\rho w = \frac{1}{r}\frac{\partial \psi}{\partial r}, \quad \rho u = -\frac{1}{r}\frac{\partial \psi}{\partial z}.$$

Using these in (28), (27) can be written

$$T\delta s = c_p\delta T + L_v\delta q + g\delta z - \left(\frac{v^2}{r} + fv\right)\delta r + \delta\left(\frac{1}{2}(u^2 + w^2)\right) + \frac{1}{\rho r}\zeta\delta\psi. \tag{30}$$

But in a steady flow, the total energy, E, is conserved, where E is defined

$$E = c_pT + L_vq + gz + \frac{1}{2}|\mathbf{V}|^2.$$

Using this in (30) gives

$$T\delta s = \delta E - \frac{1}{2}\delta v^2 - \left(\frac{v^2}{r} + fv\right)\delta r + \frac{1}{\rho r}\zeta\delta\psi. \tag{31}$$

Finally, using the definition of angular momentum per unit mass, M, we write

$$\frac{v}{r}\delta M = \frac{1}{2}\delta v^2 + \left(\frac{v^2}{r} + fv\right)\delta r,$$

so that (31) becomes

$$\delta s = \frac{1}{T}\left[\delta E - \frac{v}{r}\delta M + \frac{1}{\rho r}\zeta\delta\psi\right]. \tag{32}$$

This gives the increment of entropy between the two bounding surfaces in terms of increments of other conserved variables.

Conservation of energy, angular momentum and mass in the subcloud layer gives

$$-2\pi\psi\delta E = \pi\delta r_0^2\rho_s C_k|\mathbf{V_s}|(k_0^* - k),$$

$$2\pi\psi\delta M = \pi\delta r_0^2\rho_s C_D r_0|\mathbf{V_s}|v_s,$$

$$2\pi\delta\psi = \pi\delta r_0^2\rho_s w_0.$$

Using these in (32) and substituting the result into (26) gives an expression for the maximum surface wind speed:

$$|\mathbf{V_s}|^2 = \frac{C_k}{C_D}(k_0^* - k)\frac{T_s - T}{T} + \frac{T_s}{T}\left[v v_s\frac{r_0}{r} - \frac{\rho_0}{\rho_s}\frac{1}{\rho r}\frac{\psi\zeta w_0}{C_D|\mathbf{V_s}|}\right]. \tag{33}$$

The last term has no counterpart in the derivation of a similar equation from the dynamical equations, and represents the effect of unbalanced flow that is filtered by the balance equations. It can be shown that this last term is roughly three order-of-magnitude smaller than the other terms and thus may be neglected. The first term in brackets in (33) does not appear in (21) because of various approximations that were made in the dynamical derivation.

We see from the form of (33) (neglecting the last term) that there is a precise definition of the "outflow temperature", T_o, that makes (21) valid: It is the temperature along the angular momentum surface that passes through the locus of maximum winds, at the point at which the tangential velocity, v, vanishes. Alternatively, conservation of angular momentum gives

$$v = v_0 \frac{r_0}{r} - \frac{1}{2} f r,$$

where we have ignored the contribution of the Coriolis term at the radius of maximum winds. Subsituting this for the second term on the right of (33) and ignoring the last term, and then evaluating the resulting expression in the limit that $r \rightarrow \infty$ gives

$$|\mathbf{V_s}|^2 = \frac{C_k}{C_D}(k_0^* - k)\frac{T_s - T_o}{T_o} - \frac{T_s}{T_o}\frac{1}{2} f r_0 v_0, \tag{34}$$

where this time T_o is defined to be the environmental temperature at infinity along the streamline that originates at the locus of maximum winds. In general, the last term will be small unless the radius of maximum winds is relatively large.

4. Numerical experiments

a. Simple balance model

We first add dissipative heating to the simple, 3-layer balance model developed by Emanuel (1995b). This is an axisymmetric model phrased in potential radius coordinates, in which it is assumed that the flow is everywhere in hydrostatic and gradient wind balance. In the dynamical equations, a static stability is assumed (as in quasi-geostrophy), corresponding to moist adiabatic lapse rates of temperature along angular momentum surfaces. Moist convection is represented by assuming local thermodynamic equilibrium of the subcloud layer and by representing convective downdrafts as a function of updraft mass flux and precipitation efficiency.

The only modification we make to the model described in Emanuel (1995b) is to add a dissipative heating term of the form (6) to the subcloud layer thermodynamic equation. The effect of this on the evolution of the maximum wind speed and central surface pressure can be seen in Figure 2. The theoretical predictions of maximum wind and central pressure given by the theory of Emanuel (1995a), with and without the modification owing to dissipative heating given by (22), are also indicated. Clearly, the effect of dissipative heating is as anticipated, with a 25% increase in maximum wind and a 60% increase in the magnitude of the central pressure deficit over the environment.

b. Axisymmetric, nonhydrostatic model

We next add dissipative heating to the nonhydrostatic, axisymmetric hurricane model of Rotunno and Emanuel (1987), modified by Bister (1996). The model explicitly (albeit crudely) resolves convective clouds and is here run with a horizontal grid spacing of 7.5 km and a vertical grid spacing of 1.25 km. Turbulence in the model is represented by eddy diffusivities which depend on the local rate of strain and the Richardson number. Surface fluxes are represented by bulk aerodynamic formulae. We add dissipative heating terms of the form (4), except at the surface where it has the form (6). The results are shown in Figure 3 along with the theoretical predictions of Emanuel (1995a). Once again, the effect of dissipative heating is as anticipated. Figure 4 shows the spatial distribution of dissipative heating during the mature phase of the model storm. Clearly, the bulk of dissipative heating in the model occurs in the boundary layer.

5. Summary

Dissipative heating is almost always neglected in numerical simulations of atmospheric flows, and has until now been omitted from theories of hurricane intensity. Here we have shown that dissipative heating is by no means negligible in the case of hurricanes, increasing their kinetic energy density by roughly 50%. We therefore advocate that the dissipative heating term (4) be added to the thermodynamic equation of models used to simulate or predict hurricanes.

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Figure Captions

Figure 1: Showing the control volume (hatched) used for the derivation from conservation principles. The heavy dashed line near bottom denotes the top of the boundary layer, while the dashed line near top is an isothermal surface. The volume is bounded laterally by surfaces of constant streamfunction (ψ), angular momentum (M), total energy (E) and entropy (s).

Figure 2: Evolution with time of (**a**) the maximum winds speed (ms^{-1}) and (**b**) the central pressure (mb) in the balance model of Emanuel (1995b). The solid and dashed lines show the simulations with and without dissipative heating, respectively. Arrows at right show the theoretical maximum intensities from Emanuel (1995a).

Figure 3: Same as Figure 2, but using the nonhydrostatic model of Rotunno and Emanuel (1987). Heavy and light lines show the runs with and without dissipative heating, respectively.

Figure 4: Spatial distribution of the rate of dissipative heating ($K\ day^{-1}$) at 180 hours into the simulation using the model of Rotunno and Emanuel (1987). The graph covers the innermost 240 km of the domain, and extends upwards to 25 km altitude. The maximum point value of the dissipative heating at this time is $102K\ day^{-1}$.